TUM-HEP-321/98
IFT-15/98
CERN-TH/99-88
hep-ph/9904413

Matching Conditions for $b \to s\gamma$ and $b \to s$ *gluon* in Extensions of the Standard Model

Christoph Bobeth[1], Mikołaj Misiak[2,3,⋆] and Jörg Urban[1]

[1] *Institut für Theoretische Physik, Technische Universität Dresden, Mommsenstr. 13, D-01062 Dresden, Germany*

[2] *Physik Department, Technische Universität München, D-85748 Garching, Germany*

[3] *Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

Abstract

We evaluate matching conditions for the Wilson coefficients of operators mediating the $b \to s\gamma$ and $b \to s$ *gluon* transitions in a large class of extensions of the Standard Model. The calculation is performed at the leading order in flavour-changing couplings and includes two-loop QCD corrections. These corrections can be numerically important when approximate cancellations occur among the new physics contributions and/or the SM one.

⋆ Address in September 1997: Max Planck Institut für Physik komplexer Systeme, Nöthnitzerstr. 38, D-01197 Dresden, Germany. Permanent address: Institute of Theoretical Physics, Warsaw University, ul. Hoża 69, PL-00-681 Warsaw, Poland.

1. Introduction

The decay $B \to X_s\gamma$ is known to be a sensitive test of various new physics scenarios. Its branching ratio can significantly deviate from the Standard Model prediction in the Supersymmetric Standard Model (SSM) [1, 2], multi-Higgs-doublet models [2, 3, 4], left–right-symmetric models [5, 6, 7] and other theories [8]. Since the experimental results of CLEO [9]

$$BR[B \to X_s\gamma] = (3.15 \pm 0.35_{stat} \pm 0.32_{syst} \pm 0.26_{model}) \times 10^{-4} \tag{1}$$

and ALEPH [10]

$$BR[B \to X_s\gamma] = (3.11 \pm 0.80_{stat} \pm 0.72_{syst}) \times 10^{-4} \tag{2}$$

are consistent with the SM expectation (see below), parameter spaces of these theories get severely constrained. However, given the present sizable experimental errors, it is still conceivable that new physics effects are large, but either tend to cancel in the decay rate or tend to reverse the sign of the amplitude, when compared to the SM. In such cases, next-to-leading QCD corrections to the new physics contributions can be numerically very important [2, 4].

After the next-to-leading QCD corrections have been found [11]–[14] and confirmed [15]–[17] in the Standard Model case, all the SM predictions given in the literature fall in the range

$$BR[B \to X_s\gamma] = (3.3 \pm 0.3) \times 10^{-4}. \tag{3}$$

All the authors agree that the present theoretical uncertainty is around 10% (see e.g. [14], [17]–[20]). Even if all the authors had included the same set of corrections (which was not the case), the central values of their predictions would be allowed to differ by up to around 7%, which is the expected size of the uncalculated next-to-next-to-leading QCD corrections.

Apart from the QCD corrections, two other important contributions have been calculated recently, namely the Λ/m_c non-perturbative corrections [21] and the leading electroweak corrections [18, 22]. In both cases, the effects on the branching ratio are below the $\sim$10% overall theoretical uncertainty, but the shifts in the central value are relevant.

Theoretical analyses of $b \to s\gamma$ and $b \to s\ gluon$ are usually performed in three steps. First, the full Standard Model (or some of its extensions) is perturbatively matched on an effective theory containing only light degrees of freedom, i.e. particles much lighter

than the electroweak gauge bosons. Flavour-changing interactions in the low energy theory are mediated by effective operators of dimension higher than four. In the second step, Wilson coefficients of these operators are evolved with the help of the renormalization group equations from the electroweak scale $\mu_0 \sim M_W$ to the low-energy scale $\mu_b \sim m_b$. Finally, one evaluates matrix elements of the effective operators between the physical states of interest.

This procedure allows a resummation of large QCD logarithms such as $[\alpha_s \ln(M_W^2/m_b^2)]^n$ from all orders of the perturbation series. Moreover, its important advantage lies in that both the RGE evolution and the calculation of matrix elements are practically the same in the Standard Model and in many of its extensions. Thus, a variety of new physics effects in $B \to X_s\gamma$ can be analysed with next-to-leading accuracy without having to repeat the involved calculations of three-loop anomalous dimensions [14, 23], two-loop matrix elements [13] or Bremsstrahlung corrections [11, 15]. It is sufficient to perform the matching calculation for $b \to s\gamma$ and $b \to s$ *gluon*, including the potentially relevant next-to-leading two-loop contributions.

Performing matching calculations in theories containing exotic particles requires calculating basically the same (or at least very similar) sets of Feynman diagrams as in the SM case. The variety of final results is to a large extent due only to differences in electromagnetic charges and QCD-transformation properties of the heavy particles that are being decoupled. Consequently, it seems reasonable to first calculate the matching conditions for a relatively generic extension of the Standard Model. Results for various specific theories can then be obtained by only substituting particular values of couplings, charges and colour factors to the "generic" formulae. Presenting such "generic" formulae for the one- and two-loop matching conditions is the main purpose of the present paper.

The class of models we shall be interested in are theories in which the leading contributions to $b \to s\gamma$ and $b \to s$ *gluon* originate from (heavy fermion)–(heavy boson) loops, similarly to the SM case. Each such loop gives an additive contribution to the considered Wilson coefficient. Thus, it is sufficient to perform a calculation with only a single heavy fermion and a single heavy boson (scalar or vector). Having calculated the one-loop diagrams and two-loop gluonic corrections to them, we shall be able to reproduce the known next-to-leading matching results in the Standard Model,[2] the Two-Higgs-Doublet Model (2HDM), and gluonic parts of the NLO corrections to chargino contributions in the SSM. Examples of new results that can be obtained from our formulae are the NLO matching contributions in

[2] except for the (light quark)–(W-boson) contribution, which we calculate separately.

the left–right-symmetric models, as well as gluonic parts of the NLO corrections to neutralino and gluino contributions in the SSM.

Completing the full NLO calculation in the SSM requires, in addition, evaluating two-loop diagrams containing no gluons but only heavy particles in internal lines. Some of those results are collected in appendix C. The remaining contributions will be treated approximately in the heavy gluino case.

Our paper is organized as follows. In the next section, we evaluate leading and next-to-leading matching conditions originating from (heavy fermion)–(heavy scalar) loops. In section 3, a similar calculation is performed for (heavy fermion)–(heavy vector boson) loops. Light-quark contributions are discussed in section 4. In section 5, we specify what substitutions have to be made in the general results, for particular extensions of the SM. Section 6 is devoted to presenting a numerical example of the NLO matching contribution effects. There, we consider the SSM with decoupled gluino, which is relevant, for instance, in the context of models with gauge-mediated SUSY breaking. Appendix A summarizes several useful formulae which we have used for evaluating colour factors in the generic case. Appendix B contains a list of functions that enter our "generic" results. Finally, appendix C contains results for matching contributions originating from quartic squark vertices in the SSM.

2. Heavy fermion – heavy scalar loops

In the present section, we shall evaluate one- and two-loop matching contributions for $b \to s\gamma$ and $b \to s\ gluon$ in a theory described by the following lagrangian:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{QCD\times QED}(u,d,s,c,b) + (D^\mu\phi)^\dagger(D_\mu\phi) - m_\phi^2\phi^\dagger\phi + \bar\psi(i\not{D} - m_\psi)\psi \\ &+ \left\{C_{ijk}\phi_i^\star\bar\psi_j\left[(S_L P_L + S_R P_R)s_k + (B_L P_L + B_R P_R)b_k\right] + h.c.\right\} + \mathcal{L}_{irrelevant}. \end{aligned} \tag{4}$$

Here, $\mathcal{L}_{QCD\times QED}(u,d,s,c,b)$ denotes kinetic terms for the light quarks, photons and gluons as well as their gauge interactions. The remainder of the first line contains kinetic, gauge interaction and mass terms for a heavy complex scalar and a heavy Dirac fermion. The covariant derivatives of these fields are the following:

$$\begin{aligned} D_\mu\psi &= \left[\partial_\mu + ig_3 G_\mu^a T_{(\psi)}^a + ieQ_\psi A_\mu\right]\psi, \\ D_\mu\phi &= \left[\partial_\mu + ig_3 G_\mu^a T_{(\phi)}^a + ieQ_\phi A_\mu\right]\phi. \end{aligned} \tag{5}$$

The couplings $S_{L,R}$ and $B_{L,R}$ in eq. (4) parametrize Yukawa interactions of the heavy particles with the s- and b-quarks ($P_{L,R} = (1 \mp \gamma_5)/2$). These interaction terms must be

QED- and QCD-singlets. The heavy particles can reside in any representation of $SU(3)_{colour}$ for which such singlets exist. The following gauge-invariance constraints must be satisfied by the electric charges and colour generators:

$$\begin{aligned} Q_\psi \quad + \quad Q_\phi \quad &= -\frac{1}{3}, \\ T^a_{(\psi)jn} C_{ink} + T^a_{(\phi)in} C_{njk} &= C_{ijn} T^a_{nk}, \end{aligned} \tag{6}$$

where C_{ijk} are the Clebsch–Gordan coefficients contracting colour indices in eq. (4), and T^a on the r.h.s. above is the generator of the fundamental representation of $SU(3)$. All the generators satisfy the standard commutation relations

$$\left[T^a_{(\psi)}, T^b_{(\psi)}\right] = i f_{abc} T^c_{(\psi)}, \qquad \left[T^a_{(\phi)}, T^b_{(\phi)}\right] = i f_{abc} T^c_{(\phi)}, \qquad \left[T^a, T^b\right] = i f_{abc} T^c. \tag{7}$$

The Casimir eigenvalues for the heavy particles will be denoted by κ_ψ and κ_ϕ, respectively:

$$T^a_{(\psi)} T^a_{(\psi)} = \kappa_\psi 1, \qquad T^a_{(\phi)} T^a_{(\phi)} = \kappa_\phi 1. \tag{8}$$

We assume that the Yukawa interactions $S_{L,R}$ and $B_{L,R}$ are weak, i.e. that it is mandatory to calculate at the leading order in these interactions and α_{em}, but at the next-to-leading order in α_s. Furthermore, we assume that all the remaining interactions (denoted by $\mathcal{L}_{irrelevant}$) do not influence the $b \to s\gamma$ and $b \to s$ *gluon* amplitudes at the considered order in perturbation theory.

In each particular model, one needs to carefully verify whether the latter assumption does not lead to neglecting important contributions. Even in the Standard Model case, the top-quark Yukawa coupling Y_t and the quartic Higgs coupling λ can be similar in magnitude to the QCD gauge coupling. Potentially, the $\mathcal{O}(Y_t, \lambda)$ corrections to the considered amplitudes could be almost as important numerically as the $\mathcal{O}(\alpha_s)$ matching corrections. However, it has been explicitly checked in ref. [22] that this is not the case for $b \to s\gamma$, i.e. that $\mathcal{O}(Y_t, \lambda)$ effects in the SM are only around 1% in the branching ratio, which is partly due to accidental cancellations.

In the present and in the following three sections of this article, we shall neglect $\mathcal{L}_{irrelevant}$, i.e. we shall restrict ourselves only to the leading one-loop diagrams, and to the next-to-leading two-loop diagrams *with gluons*.[3] Such diagrams involving (heavy fermion)–(heavy scalar) loops are presented in figs. 1 and 2, respectively. Small circles denote places where the

[3] In section 6, where the SSM case will be considered numerically, we shall include also flavour-conserving gluino couplings and the quartic squark couplings, which are proportional to α_s.

external gluon can couple in the $b \to s$ *gluon* case. The external photon in the $b \to s\gamma$ case can couple in the same places, except the ones denoted by "5" on internal gluon propagators. Thus, we have 45 two-loop diagrams in the $b \to s$ *gluon* case, and 37 two-loop diagrams in the $b \to s\gamma$ case.

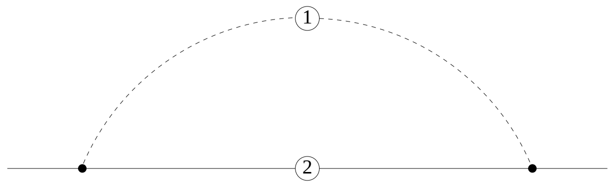

Figure 1: One-loop diagrams

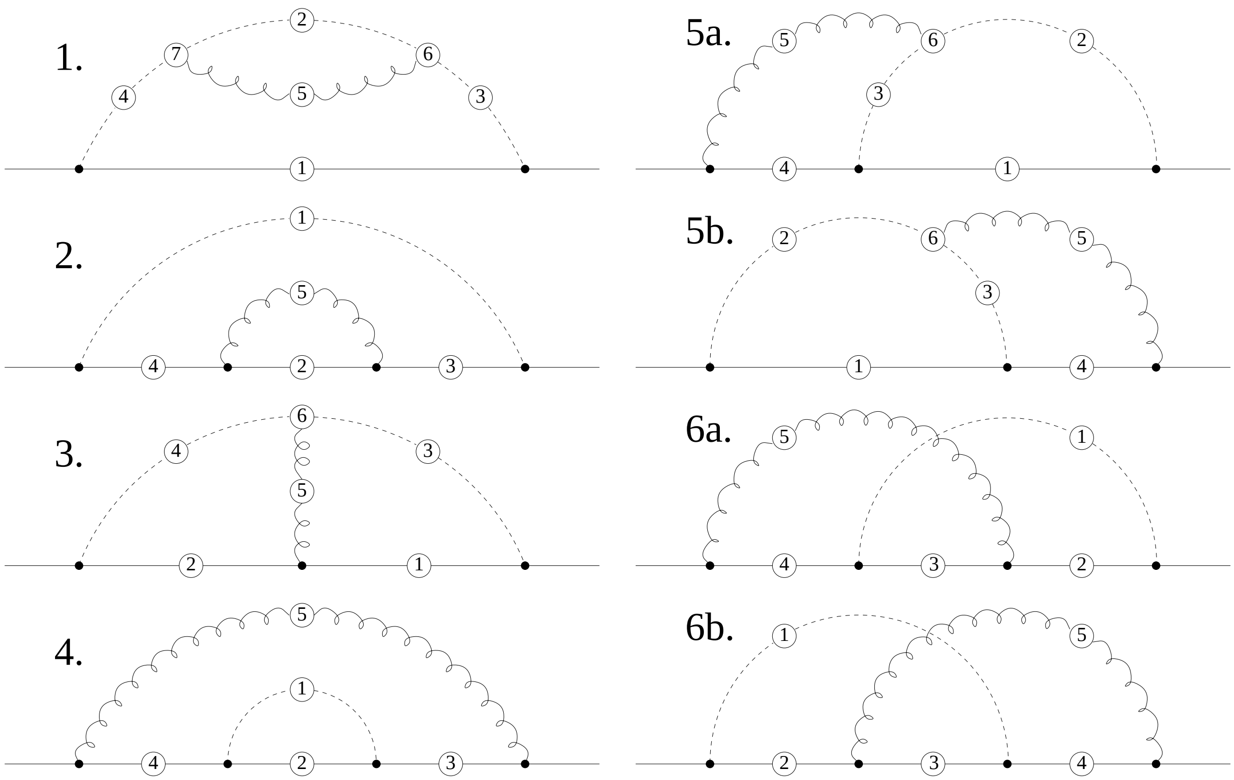

Figure 2: Two-loop diagrams

We evaluate all the diagrams off shell, in the 't Hooft–Feynman version of the background field gauge for QCD. For all the quantities we need to renormalize, we use the $\overline{MS}$ scheme with the renormalization scale μ_0 that is assumed to be of the same order as the heavy masses. Before performing the momentum integration, we expand the integrands up to second order in (external momenta)/(heavy masses). All the spurious IR divergences arising in this procedure are regularized dimensionally.

Next, we require equality of our result to the similar off-shell 1PI Green function in the effective theory described by the following effective lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}_{QCD\times QED}(u,d,s,c,b) \;+\; \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[\sum_{i=4,7,8} (C_i P_i + C_i' P_i') \;+\; \left(\begin{array}{c} \text{EOM-vanishing} \\ \text{operators} \end{array} \right) \right], \tag{9}$$

where

$$\begin{aligned} P_4 &= (\bar{s}\gamma_\mu T^a P_L b) \textstyle\sum_q (\bar{q}\gamma^\mu T^a q), \\ P_7 &= \tfrac{e}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu}, \\ P_8 &= \tfrac{g_3}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} T^a P_R b) G^a_{\mu\nu}, \end{aligned} \tag{10}$$

and the primed operators P_i' are obtained from the ones above by changing $P_{L,R}$ to $P_{R,L}$. The structure of the EOM-vanishing operators (i.e. operators that vanish by the QCD$\times$QED equations of motion) as well as other elements of our matching procedure can be found in ref. [24] where the Standard Model case is described in detail.[4]

The obtained Wilson coefficients

$$C_i(\mu_0) = C_i^{(0)}(\mu_0) + \frac{\alpha_s(\mu_0)}{4\pi} C_i^{(1)}(\mu_0) + ... \qquad (i = 4,7,8) \tag{11}$$

(as well as their primed counterparts) can be written as linear combinations of various functions of $x = (m_\psi/m_\phi)^2$:

$$C_4^{(0)}(\mu_0) = 0, \tag{12}$$

$$\begin{aligned} C_7^{(0)}(\mu_0) = \; & N \left\{ R_1 \left[g_1(x) Q_\psi - \tfrac{1}{x} g_1\left(\tfrac{1}{x}\right) Q_\phi \right] \right. \\ & \left. + R_2 \left[g_2(x) Q_\psi + \left(x g_2(x) + \tfrac{1}{2} \right) Q_\phi \right] \right\}, \end{aligned} \tag{13}$$

$$\begin{aligned} C_8^{(0)}(\mu_0) = \; & N \left\{ R_1 \left[\tfrac{3}{8}(\kappa_\psi - \kappa_\phi) \left(g_1(x) + \tfrac{1}{x} g_1\left(\tfrac{1}{x}\right) \right) + \tfrac{1}{2} g_1(x) - \tfrac{1}{2x} g_1\left(\tfrac{1}{x}\right) \right] \right. \\ & \left. + R_2 \left[\tfrac{3}{8}(\kappa_\psi - \kappa_\phi) \left((1-x) g_2(x) - \tfrac{1}{2} \right) + \tfrac{x+1}{2} g_2(x) + \tfrac{1}{4} \right] \right\}, \end{aligned} \tag{14}$$

$$C_4^{(1)}(\mu_0) = N S_L^* B_L \left[(\kappa_\phi - \kappa_\psi) f_1(x) + f_2(x) \right], \tag{15}$$

$$\begin{aligned} C_7^{(1)}(\mu_0) = \; & 3 \ln \tfrac{\mu_0^2}{m_\phi^2} \left[(2\kappa_\psi - \kappa_\phi) x \left(\frac{\partial C_7^{(0)}}{\partial x} \right)_{S,B} + \left(\tfrac{28}{9} + \kappa_\psi - \kappa_\phi \right) C_7^{(0)} + (\kappa_\psi - \tfrac{4}{3}) C_7^{(0)LR} - \tfrac{16}{27} C_8^{(0)} \right] \\ & + N \left\{ R_1 \left[\left(h_1(x)\kappa_\psi + h_2(x)\kappa_\phi + h_3(x) \right) Q_\psi + \left(h_4(x)\kappa_\psi + h_5(x)\kappa_\phi + h_6(x) \right) Q_\phi \right] \right. \\ & \left. + R_2 \left[\left(h_7(x)\kappa_\psi + h_8(x)\kappa_\phi + h_9(x) \right) Q_\psi + \left(h_{10}(x)\kappa_\psi + h_{11}(x)\kappa_\phi + h_{12}(x) \right) Q_\phi \right] \right\}, \end{aligned} \tag{16}$$

$$\begin{aligned} C_8^{(1)}(\mu_0) = \; & 3 \ln \tfrac{\mu_0^2}{m_\phi^2} \left[(2\kappa_\psi - \kappa_\phi) x \left(\frac{\partial C_8^{(0)}}{\partial x} \right)_{S,B} + \left(\tfrac{26}{9} + \kappa_\psi - \kappa_\phi \right) C_8^{(0)} + (\kappa_\psi - \tfrac{4}{3}) C_8^{(0)LR} \right] \\ & + N \left\{ R_1 \left[h_{13}(x)\kappa_\psi^2 + h_{14}(x)\kappa_\phi^2 + h_{15}(x)\kappa_\psi\kappa_\phi + h_{16}(x)\kappa_\psi + h_{17}(x)\kappa_\phi + h_{18}(x) \right] \right. \\ & \left. + R_2 \left[h_{19}(x)\kappa_\psi^2 + h_{20}(x)\kappa_\phi^2 + h_{21}(x)\kappa_\psi\kappa_\phi + h_{22}(x)\kappa_\psi + h_{23}(x)\kappa_\phi + h_{24}(x) \right] \right\} \end{aligned} \tag{17}$$

[4] Four-fermion operators generated by the photon exchange (such as $(\bar{s}\gamma_\mu P_L b)(\bar{e}\gamma^\mu e)$) are ignored here but included in ref. [24]. They are irrelevant to $b \to s\gamma$ and $b \to s\ gluon$, but they matter, for instance, in $b \to s e^+ e^-$.

with

$$R_1 = S_L^* B_L + \frac{m_s}{m_b} S_R^* B_R \qquad \text{and} \qquad R_2 = S_L^* B_R \frac{m_\psi}{m_b}. \tag{18}$$

The primed Wilson coefficients can be obtained from the ones above by simply interchanging the chirality subscripts $L \leftrightarrow R$. Explicit expressions for the functions $f_1(x)$, $f_2(x)$, $g_1(x)$, $g_2(x)$ and $h_1(x)$–$h_{24}(x)$ are given in appendix B.

The symbols $C_i^{(0)LR}$ used in the latter two equations denote those parts of the leading-order coefficients $C_i^{(0)}$ that are proportional to $S_L^* B_R$. The global normalization constant N equals to

$$N = \frac{\xi\sqrt{2}}{8 m_\phi^2 G_F V_{ts}^* V_{tb}}, \tag{19}$$

where ξ parametrizes the contraction of the Clebsch–Gordan coefficients:

$$C_{ijk}^* C_{ijn} = \xi \delta_{kn}. \tag{20}$$

The subscripts "S, B" at the derivatives of $C_7^{(0)}$ and $C_8^{(0)}$ mean that the products $S_L^* B_L$, $S_R^* B_R$ and $S_L^* B_R \frac{m_\psi}{m_b}$ need to be treated as independent of x when these derivatives are taken.

It is remarkable that colour factors of the two-loop diagrams could have been reduced to quite a simple form, even though we have not specified the representations of $SU(3)_{colour}$ in which the heavy particles reside. Several identities that are useful in performing this reduction are summarized in appendix A.

3. Heavy fermion – heavy vector boson loops

Here, we are going to consider only QCD-singlet vector bosons V_μ, and heavy fermions ψ in the fundamental representation of $SU(3)$. The lagrangian is now assumed to have the following form:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{QCD\times QED}(u,d,s,c,b) - (D_\mu V_\nu)^*(D^\mu V^\nu) + m_V^2 V_\mu^* V^\mu + ie\omega V_\mu^* V_\nu F^{\mu\nu} + \bar{\psi}(i\not{D} - m_\psi)\psi \\ &+ \left\{ V_\mu^* \bar{\psi}\gamma^\mu \left[(\sigma_L P_L + \sigma_R P_R)s + (\beta_L P_L + \beta_R P_R)b \right] + h.c. \right\} + \mathcal{L}_{irrelevant}, \end{aligned} \tag{21}$$

where $F^{\mu\nu}$ is the electromagnetic field-strength tensor, and

$$D_\mu V_\nu = (\partial_\mu + ieQ_V A_\mu) V_\nu \qquad \text{with} \qquad Q_V = -Q_\psi - \frac{1}{3}. \tag{22}$$

The constants $\sigma_{L,R}$ and $\beta_{L,R}$ are some arbitrary weak coupling constants. If we substitute in eq. (21) $V \to W$, $\psi \to t$, $Q_V \to -1$, $\omega \to -2$, $\sigma_R, \beta_R \to 0$, $\sigma_L \to -\frac{g_2}{\sqrt{2}} V_{ts}$ and $\beta_L \to -\frac{g_2}{\sqrt{2}} V_{tb}$, we obtain all the Standard Model interactions that are relevant to the (W-boson)–(top

quark) loop contributions to $b \to s\gamma$ in the 't Hooft–Feynman version of the background field gauge.

The present calculation differs from the Standard Model one mainly by that it allows right-handed couplings of the heavy vector boson to fermions. Such couplings occur for the W-boson in the left–right-symmetric models. Their small magnitude can be compensated by the large ratio m_t/m_b in contributions to $B \to X_s\gamma$ [5, 6].

The diagrams we need to consider now can be obtained from the ones presented in figs. 1 and 2 by removing all the diagrams with gluon–scalar couplings and replacing the scalar by the vector boson. In this way, we obtain 16 two-loop diagrams in both the $b \to s$ *gluon* and $b \to s\gamma$ cases.

The effective lagrangian with which our full theory is being matched remains the same as in eq. (9). The contributions to the Wilson coefficients we obtain in the present case read

$$
\begin{aligned}
C_4^{(0)}(\mu_0) &= 0, & (23)\\
C_7^{(0)}(\mu_0) &= \tilde{N}\left[\left(\sigma_L^*\beta_L + \frac{m_s}{m_b}\sigma_R^*\beta_R\right)(j_1(x)Q_\psi + j_2(x)Q_V + j_3(x)\omega)\right.\\
&\quad \left. +\sigma_L^*\beta_R\frac{m_\psi}{m_b}(j_4(x)(Q_\psi + Q_V) + j_5(x)\omega)\right], & (24)\\
C_8^{(0)}(\mu_0) &= \tilde{N}\left[\left(\sigma_L^*\beta_L + \frac{m_s}{m_b}\sigma_R^*\beta_R\right) j_1(x) + \sigma_L^*\beta_R\frac{m_\psi}{m_b} j_4(x)\right], & (25)\\
C_4^{(1)}(\mu_0) &= \tilde{N}\sigma_L^*\beta_L\left[\frac{-12x^2+18x-4}{3(1-x)^4}\ln x + \frac{-25x^2+29x+2}{9(1-x)^3}\right], & (26)\\
C_7^{(1)}(\mu_0) &= \ln\frac{\mu_0^2}{m_V^2}\left[8x\left(\frac{\partial C_7^{(0)}}{\partial x}\right)_{\sigma,\beta} + \frac{16}{3}C_7^{(0)} - \frac{16}{9}C_8^{(0)}\right]\\
&\quad +\tilde{N}\left[\left(\sigma_L^*\beta_L + \frac{m_s}{m_b}\sigma_R^*\beta_R\right)(k_1(x)Q_\psi + k_2(x)Q_V + k_3(x)\omega)\right.\\
&\quad \left. +\sigma_L^*\beta_R\frac{m_\psi}{m_b}(k_4(x)Q_\psi + k_5(x)Q_V + k_6(x)\omega)\right], & (27)\\
C_8^{(1)}(\mu_0) &= \ln\frac{\mu_0^2}{m_V^2}\left[8x\left(\frac{\partial C_8^{(0)}}{\partial x}\right)_{\sigma,\beta} + \frac{14}{3}C_8^{(0)}\right]\\
&\quad +\tilde{N}\left[\left(\sigma_L^*\beta_L + \frac{m_s}{m_b}\sigma_R^*\beta_R\right) k_7(x) + \sigma_L^*\beta_R\frac{m_\psi}{m_b} k_8(x)\right]. & (28)
\end{aligned}
$$

As before, contributions to the primed coefficients can be obtained by simply interchanging the L and R subscripts in the couplings σ and β. The normalization constant $\tilde{N}$ reads

$$\tilde{N} = \frac{\sqrt{2}}{8m_V^2 G_F V_{ts}^* V_{tb}}. \tag{29}$$

The functions $j_1(x)$–$j_5(x)$ and $k_1(x)$–$k_8(x)$ of $x = (m_\psi/m_V)^2$ are given explicitly in appendix B.

4. Contributions from (W-boson)–(light quark) loops

In order to obtain complete matching results, we need to consider contributions from loops containing light quarks. In the Standard Model case, loops with W-bosons and charm-quarks are relevant. Diagrams with up-quarks are analogous, but numerically less important because of the smallness of the CKM matrix element V_{ub}.

In the following, we shall assume that only the Standard Model contributions are non-negligible in the light-quark case. This is a correct assumption in many extensions of the Standard Model (e.g. 2HDM or SSM) in which couplings of heavy scalars to light fermions are suppressed by small Yukawa couplings. In the left–right-symmetric models, there are additional contributions from right-handed coupling of the W-boson to quarks. However, for the light quarks, such contributions to $b \to s\gamma$ and $b \to s$ *gluon* are *not* enhanced by the large ratio m_t/m_b. Consequently, they are negligible when compared to the top-quark one.[5]

The Standard Model diagrams we need to consider here are the same as in the previous section, with the heavy fermion replaced by the light quark. However, the effective theory side is now somewhat more complicated, since we need to include operators that contain light quarks. In the charm-quark case, they read

$$\begin{aligned} P_1 = &\ (\bar{s}\gamma_\mu P_L T^a c)(\bar{c}\gamma^\mu P_L T^a b), \\ P_2 = &\ (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma^\mu P_L b). \end{aligned} \tag{30}$$

Their Wilson coefficients are found by considering the $\bar{s}b\bar{c}c$ Green function[6]

$$\begin{aligned} &C_1^{(0)}(\mu_0) = 0, && C_2^{(0)}(\mu_0) = r_{ct}, \\ &C_1^{(1)}(\mu_0) = \left[-15 - 6\ln\frac{\mu_0^2}{M_W^2}\right] r_{ct}, && C_2^{(1)}(\mu_0) = 0. \end{aligned} \tag{31}$$

where $r_{ct} = V_{cs}^* V_{cb}/V_{ts}^* V_{tb}$. The value of $C_1^{(1)}$ has been obtained in the $\overline{MS}$ scheme (applied throughout this paper) and using the evanescent operator

$$E_1 = (\bar{s}\gamma_\mu\gamma_\nu\gamma_\rho P_L T^a c)(\bar{c}\gamma^\mu\gamma^\nu\gamma^\rho P_L T^a b) - 16 P_1. \tag{32}$$

After performing the matching for P_1 and P_2, contributions to the Wilson coefficients of P_4, P_7 and P_8 can be found. We obtain

$$\delta^c C_4^{(0)}(\mu_0) = 0, \qquad \delta^c C_4^{(1)}(\mu_0) = \left(\frac{7}{9} - \frac{2}{3}\ln\frac{\mu_0^2}{M_W^2}\right) r_{ct}, \tag{33}$$

[5] Unless the left- and right-handed CKM matrices have a very different hierarchy of their elements, which we assume not to be the case here.

[6] See ref. [24] for more details.

$$\delta^c C_7^{(0)}(\mu_0) = \tfrac{23}{36} r_{ct}, \qquad \delta^c C_7^{(1)}(\mu_0) = \left(-\tfrac{713}{243} - \tfrac{4}{81}\ln\tfrac{\mu_0^2}{M_W^2}\right) r_{ct}, \tag{34}$$

$$\delta^c C_8^{(0)}(\mu_0) = \tfrac{1}{3} r_{ct}, \qquad \delta^c C_8^{(1)}(\mu_0) = \left(-\tfrac{91}{324} + \tfrac{4}{27}\ln\tfrac{\mu_0^2}{M_W^2}\right) r_{ct}. \tag{35}$$

Results for the up-quark loops are obtained from the above ones by replacing r_{ct} by $r_{ut} = V_{us}^* V_{ub}/V_{ts}^* V_{tb}$. After adding the up- and charm-quark contributions, we can make use of the equality $r_{ut} + r_{ct} = -1$, which follows from unitarity of the CKM matrix.

The primed coefficients are negligible in the Standard Model case, because they are proportional to small Yukawa couplings of the would-be Goldstone boson to light quarks.

5. Substitutions

Let us first use our generic results to reproduce the known SM matching conditions at the next-to-leading order in QCD. The only scalar we need to consider in the SM is the charged would-be Goldstone boson. We use the 't Hooft–Feynman version of the background field gauge, in which the (W-boson)-photon-scalar vertices are absent. The only heavy fermion is the top quark, for which one finds

$$N\left(S_L^* B_L + \frac{m_s}{m_b} S_R^* B_R\right) = \frac{x}{2}\left(1 + \frac{m_s^2}{M_W^2}\right) \simeq \frac{x}{2} \qquad \text{with} \quad x = \frac{m_t^2}{M_W^2}, \tag{36}$$

$$N S_L^* B_R \frac{m_\psi}{m_b} = -\frac{x}{2}, \tag{37}$$

$$\tilde{N} \sigma_L^* \beta_L = \frac{1}{2} \qquad \text{and} \quad \sigma_R = \beta_R = 0. \tag{38}$$

The remaining substitutions one needs to make are $m_\psi \to m_t$, $m_\phi, m_V \to M_W$, $Q_\psi \to \frac{2}{3}$, $Q_\phi, Q_V \to -1$, $\omega \to -2$, $\kappa_\psi \to \frac{4}{3}$ and $\kappa_\phi \to 0$. Adding up the results from sections 2, 3 and 4, one easily finds

$$C_{7,8}^{(0)}(\mu_0) = -\frac{3}{2} x H_1^{[7,8]}(x) \qquad \text{with} \qquad \begin{cases} H_1^{[7]}(x) = \frac{-3x^2+2x}{6(1-x)^4}\ln x + \frac{-8x^2-5x+7}{36(1-x)^3} \\[2ex] H_1^{[8]}(x) = \frac{x}{2(1-x)^4}\ln x + \frac{-x^2+5x+2}{12(1-x)^3} \end{cases} \tag{39}$$

$$C_4^{(1)}(\mu_0) = \tfrac{4x^4-16x^3+15x^2}{6(1-x)^4}\ln x + \tfrac{7x^3-35x^2+42x-8}{12(1-x)^3} + \tfrac{2}{3}\ln\tfrac{\mu_0^2}{m_t^2}, \tag{40}$$

$$\begin{aligned} C_7^{(1)}(\mu_0) = {}& \tfrac{-16x^4-122x^3+80x^2-8x}{9(1-x)^4} Li_2\left(1-\tfrac{1}{x}\right) + \tfrac{-4x^5+407x^4+1373x^3-957x^2+45x}{81(1-x)^5}\ln x \\ & + \tfrac{1520x^4+12961x^3-12126x^2+3409x-580}{486(1-x)^4} + \left[\tfrac{6x^4+46x^3-28x^2}{3(1-x)^5}\ln x + \tfrac{106x^4+287x^3+1230x^2-1207x+232}{81(1-x)^4}\right]\ln\tfrac{\mu_0^2}{m_t^2}, \end{aligned} \tag{41}$$

$$\begin{aligned} C_8^{(1)}(\mu_0) = {}& \tfrac{-4x^4+40x^3+41x^2+x}{6(1-x)^4} Li_2\left(1-\tfrac{1}{x}\right) + \tfrac{32x^5-16x^4-2857x^3-3981x^2-90x}{216(1-x)^5}\ln x \\ & + \tfrac{611x^4-13346x^3-29595x^2+1510x-652}{1296(1-x)^4} + \left[\tfrac{-17x^3-31x^2}{2(1-x)^5}\ln x + \tfrac{89x^4-446x^3-1437x^2-950x+152}{108(1-x)^4}\right]\ln\tfrac{\mu_0^2}{m_t^2}. \end{aligned} \tag{42}$$

These results agree with the main findings of refs. [12, 16, 17]. Contributions to the primed coefficients can be neglected in the SM, because they are suppressed by m_s/m_b.

Let us now turn to the Two-Higgs-Doublet Model II. For diagrams with the physical charged higgson exchanges, the necessary substitutions in the formulae of section 2 are the following: $m_\psi \to m_t$, $m_\phi \to M_{H^-}$, $Q_\psi \to \frac{2}{3}$, $Q_\phi \to -1$, $\kappa_\psi \to \frac{4}{3}$, $\kappa_\phi \to 0$ and

$$N\left(S_L^* B_L + \frac{m_s}{m_b} S_R^* B_R\right) = \frac{y}{2}\cot^2\beta\left(1 + \frac{m_s^2}{M_W^2}\right) \simeq \frac{y}{2}\cot^2\beta, \tag{43}$$

$$N S_L^* B_R \frac{m_\psi}{m_b} = \frac{y}{2} \qquad \text{with} \quad y = \frac{m_t^2}{M_{H^-}^2}. \tag{44}$$

The resulting contributions to the Wilson coefficients read

$$\delta^{H^-} C_{7,8}^{(0)}(\mu_0) = -\frac{y\cot^2\beta}{2} H_1^{[7,8]}(y) + \frac{1}{2} H_2^{[7,8]}(y) \qquad \text{with} \quad \begin{cases} H_2^{[7]}(y) = \frac{-3y^2+2y}{3(1-y)^3}\ln y + \frac{-5y^2+3y}{6(1-y)^2} \\ H_2^{[8]}(y) = \frac{y}{(1-y)^3}\ln y + \frac{-y^2+3y}{2(1-y)^2} \end{cases} \tag{45}$$

$$\delta^{H^-} C_4^{(1)}(\mu_0) = \cot^2\beta\left[\frac{3y^2-2y}{6(1-y)^4}\ln y + \frac{-7y^3+29y^2-16y}{36(1-y)^3}\right], \tag{46}$$

$$\begin{aligned}\delta^{H^-} C_7^{(1)}(\mu_0) = {} & \cot^2\beta\left\{\frac{16y^4-74y^3+36y^2}{9(1-y)^4} Li_2\left(1-\frac{1}{y}\right) + \frac{-63y^4+807y^3-463y^2+7y}{81(1-y)^5}\ln y\right. \\ & + \left.\frac{-1202y^4+7569y^3-5436y^2+797y}{486(1-y)^4} + \left[\frac{6y^4+46y^3-28y^2}{9(1-y)^5}\ln y + \frac{-14y^4+135y^3-18y^2-31y}{27(1-y)^4}\right]\ln\frac{\mu_0^2}{m_t^2}\right\} \\ & + \frac{-32y^3+112y^2-48y}{9(1-y)^3} Li_2\left(1-\frac{1}{y}\right) + \frac{14y^3-128y^2+66y}{9(1-y)^4}\ln y + \frac{8y^3-52y^2+28y}{3(1-y)^3} \\ & + \left[\frac{-12y^3-56y^2+32y}{9(1-y)^4}\ln y + \frac{16y^3-94y^2+42y}{9(1-y)^3}\right]\ln\frac{\mu_0^2}{m_t^2},\end{aligned} \tag{47}$$

$$\begin{aligned}\delta^{H^-} C_8^{(1)}(\mu_0) = {} & \cot^2\beta\left\{\frac{13y^4-17y^3+30y^2}{6(1-y)^4} Li_2\left(1-\frac{1}{y}\right) + \frac{-468y^4+321y^3-2155y^2-2y}{216(1-y)^5}\ln y\right. \\ & + \left.\frac{-4451y^4+7650y^3-18153y^2+1130y}{1296(1-y)^4} + \left[\frac{-17y^3-31y^2}{6(1-y)^5}\ln y + \frac{-7y^4+18y^3-261y^2-38y}{36(1-y)^4}\right]\ln\frac{\mu_0^2}{m_t^2}\right\} \\ & + \frac{-17y^3+25y^2-36y}{6(1-y)^3} Li_2\left(1-\frac{1}{y}\right) + \frac{34y^3-7y^2+165y}{12(1-y)^4}\ln y + \frac{29y^3-44y^2+143y}{8(1-y)^3} \\ & + \left[\frac{17y^2+19y}{3(1-y)^4}\ln y + \frac{7y^3-16y^2+81y}{6(1-y)^3}\right]\ln\frac{\mu_0^2}{m_t^2}.\end{aligned} \tag{48}$$

These results are in agreement with refs. [2, 4]. Similarly to the SM case, the primed coefficients are suppressed by m_s/m_b, and can be neglected.

The third example we would like to discuss here is the $SU(2)_L \times SU(2)_R \times U(1)$ model. The notation introduced in ref. [5] will be followed. We shall restrict ourselves to such contributions from the light W-boson (and the corresponding would-be Goldstone boson) which are suppressed by the small W-boson mixing angle ζ, but simultaneously enhanced by the large quark-mass ratio $\frac{m_t}{m_b}$. In this case, the substitutions one needs to make in the results of sections 2 and 3 are the following:

$$\begin{aligned} & N S_L^* B_R \tfrac{m_\psi}{m_b} = \tfrac{x}{2} A^{tb}, && \tilde{N}\sigma_L^* \beta_R \tfrac{m_\psi}{m_b} = \tfrac{1}{2} A^{tb}, \\ & N S_R^* B_L \tfrac{m_\psi}{m_b} = \tfrac{x}{2} \left(A^{ts}\right)^*, && \tilde{N}\sigma_R^* \beta_L \tfrac{m_\psi}{m_b} = \tfrac{1}{2} \left(A^{ts}\right)^*, \end{aligned} \tag{49}$$

where $x = \frac{m_t^2}{M_W^2}$ and

$$A^{tb(s)} = \frac{m_t}{m_b}\, \zeta e^{i\alpha}\, \frac{g_{2R} V^R_{tb(s)}}{g_{2L} V^L_{tb(s)}} \;\; + \;\; \mathcal{O}(\zeta^2). \tag{50}$$

The "left–left" and "right–right" products of couplings can be set to zero now, because their non-SM parts are of order $\mathcal{O}(\zeta^2, \frac{m_b}{M_W})$, i.e. negligibly small. Substitutions for masses, charges and colour factors here are the same as for top-quark loops in the Standard Model case.

The obtained contributions to the Wilson coefficients read

$$\delta^{LR} C_4(\mu_0) \;=\; 0, \tag{51}$$

$$\delta^{LR} C_7^{(0)}(\mu_0) \;=\; A^{tb} \left[\tfrac{3x^2-2x}{2(1-x)^3} \ln x \;+\; \tfrac{-5x^2+31x-20}{12(1-x)^2} \right], \tag{52}$$

$$\delta^{LR} C_8^{(0)}(\mu_0) \;=\; A^{tb} \left[\tfrac{-3x}{2(1-x)^3} \ln x \;+\; \tfrac{-x^2-x-4}{4(1-x)^2} \right], \tag{53}$$

$$\begin{aligned} \delta^{LR} C_7^{(1)}(\mu_0) \;=\; & A^{tb} \left\{ \tfrac{-32x^3-112x^2+48x}{9(1-x)^3} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{86x^3+120x^2-30x-32}{9(1-x)^4} \ln x \right. \\ & \left. +\; \tfrac{24x^3+320x^2-220x+20}{9(1-x)^3} \;+\; \left[\tfrac{12x^3+56x^2-32x}{3(1-x)^4} \ln x \;+\; \tfrac{16x^3+90x^2+66x-64}{9(1-x)^3} \right] \ln \tfrac{\mu_0^2}{m_t^2} \right\}, \end{aligned} \tag{54}$$

$$\begin{aligned} \delta^{LR} C_8^{(1)}(\mu_0) \;=\; & A^{tb} \left\{ \tfrac{-17x^3+89x^2+12x}{6(1-x)^3} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{34x^3-375x^2-207x-28}{12(1-x)^4} \ln x \right. \\ & \left. +\; \tfrac{87x^3-640x^2-451x-148}{24(1-x)^3} \;+\; \left[\tfrac{-17x^2-19x}{(1-x)^4} \ln x \;+\; \tfrac{7x^3-36x^2-159x-28}{6(1-x)^3} \right] \ln \tfrac{\mu_0^2}{m_t^2} \right\}. \end{aligned} \tag{55}$$

The primed coefficients are obtained from the above ones by changing A^{tb} to $(A^{ts})^*$. Expressions for the leading-order coefficients are in agreement with ref. [5]. The next-to-leading results are new.

Physical charged scalars present in the left–right-symmetric models can give contributions enhanced by $\frac{m_t}{m_b}$, too [7]. Such contributions can be calculated with the help of our generic formulae as well. However, their explicit form and potential numerical importance depend on details of the Higgs sector that is not unique in these models.

Finally, let us turn to the Supersymmetric Standard Model. In this case, we shall *exactly* follow the notation of ref. [25].[7] In particular, the relevant couplings of charginos, neutralinos and gluinos to matter will be parametrized by $X^{U_{L,R}}$, $Z^{D_{L,R}}$ and $g_3\Gamma^{D_{L,R}}$, respectively. However, we shall formally treat all these matrices as independent from other parameters of the model. Only diagrams containing gluons have been included in their one-loop QCD renormalization, and we have used the $\overline{MS}$ scheme in dimensional regularization (not in dimensional reduction). In effect, immediate substitution of tree-level supersymmetric expressions for these matrices (eq. (2.12) of ref. [25]) is allowed only for the leading-order terms

[7] The CKM matrix will be denoted by K in all the SSM formulae.

in the results presented below. Section 6 contains a discussion of applicability and treatment of the $\mathcal{O}(\alpha_s)$ terms.

In the results presented below, flavour-conserving gluino interactions as well as strong quartic squark interactions have been formally treated as the weak ones. Additional contributions to the considered Wilson coefficients from two-loop diagrams with strong quartic squark interactions are collected in appendix C. A complete inclusion of strong gluino coupling effects at NLO is beyond the scope of the present paper. Consequently, our results can be used for making numerical predictions in the SSM only in the heavy-gluino limit (see section 6).

At one loop in the SSM, $b \to s\gamma$ and $b \to s$ *gluon* off-shell Green functions receive contributions from chargino–squark, neutralino–squark and gluino–squark loops, in addition to the SM and charged higgson contributions that have been already discussed. For the loops containing chargino $\tilde{\chi}_I^-$ $(I = 1, 2)$ and the up-squark $\tilde{u}_A$ $(A = 1, ..., 6)$, as well as for the gluonic corrections to them, we need to make the following substitutions in section 2: $m_\psi \to m_{\tilde{\chi}_I^-}$, $m_\phi \to m_{\tilde{u}_A}$, $Q_\psi \to -1$, $Q_\phi \to 2/3$, $\kappa_\psi \to 0$, $\kappa_\phi \to 4/3$ and

$$
\begin{aligned}
N\left(S_L^* B_L + \frac{m_s}{m_b} S_R^* B_R\right) &= \frac{1}{g_2^2 K_{ts}^* K_{tb}} \frac{M_W^2}{m_{\tilde{u}_A}^2} \left(X_I^{U_L}\right)^*_{A2} \left(X_I^{U_L}\right)_{A3} + \mathcal{O}\left(\frac{m_s^2}{M_W^2}\right), \\
N S_L^* B_R \frac{m_\psi}{m_b} &= \frac{1}{g_2^2 K_{ts}^* K_{tb}} \frac{M_W^2}{m_{\tilde{u}_A}^2} \left(X_I^{U_L}\right)^*_{A2} \left(X_I^{U_R}\right)_{A3} \frac{m_{\tilde{\chi}_I^-}}{m_b}. \qquad (56)
\end{aligned}
$$

After summing up all the chargino and up-squark species, we obtain the following contributions to the Wilson coefficients

$$
\delta^{\tilde{\chi}^-} C_4^{(1)}(\mu_0) = \frac{1}{g_2^2 K_{ts}^* K_{tb}} \sum_{A=1}^{6} \sum_{I=1}^{2} \frac{M_W^2}{m_{\tilde{\chi}_I^-}^2} \left(X_I^{U_L}\right)^*_{A2} \left(X_I^{U_L}\right)_{A3} H_1^{[4]}(z_{AI}) \qquad \text{with} \quad z_{AI} = \frac{m_{\tilde{u}_A}^2}{m_{\tilde{\chi}_I^-}^2}, \qquad (57)
$$

$$
\begin{aligned}
\delta^{\tilde{\chi}^-} C_{7,8}(\mu_0) &= \frac{1}{g_2^2 K_{ts}^* K_{tb}} \sum_{A=1}^{6} \sum_{I=1}^{2} \frac{M_W^2}{m_{\tilde{\chi}_I^-}^2} \times \\
&\times \Bigg\{ \left(X_I^{U_L}\right)^*_{A2} \left(X_I^{U_L}\right)_{A3} \left[H_1^{[7,8]}(z_{AI}) + \frac{\alpha_s}{4\pi} \left(H_1^{[7,8]\prime}(z_{AI}) + H_1^{[7,8]\prime\prime}(z_{AI}) \ln\left(\frac{\mu_0^2}{m_{\tilde{u}_A}^2}\right)\right)\right] \\
&+ \frac{m_{\tilde{\chi}_I^-}}{m_b} \left(X_I^{U_L}\right)^*_{A2} \left(X_I^{U_R}\right)_{A3} \left[H_2^{[7,8]}(z_{AI}) + \lambda^{[7,8]} + \frac{\alpha_s}{4\pi} \left(H_2^{[7,8]\prime}(z_{AI}) + H_2^{[7,8]\prime\prime}(z_{AI}) \ln\left(\frac{\mu_0^2}{m_{\tilde{u}_A}^2}\right)\right)\right] \Bigg\}. \qquad (58)
\end{aligned}
$$

The constants $\lambda^{[7]} = \frac{5}{6}$ and $\lambda^{[8]} = \frac{1}{2}$ drop out by unitarity of squark mixing matrices when the chargino–matter couplings $X^{U_{L,R}}$ are no longer treated as arbitrary but expressed in terms of other SSM parameters. The functions $H_{1,2}^{[7,8]}$ have already been encountered in the

SM and 2HDM cases. The functions $H_1^{[4]}$, $H_{1,2}^{[7,8]\prime}$ and $H_{1,2}^{[7,8]\prime\prime}$ have the following explicit form:

$$
\begin{aligned}
H_1^{[4]}(x) &= \tfrac{-1}{3(1-x)^4}\ln x \;+\; \tfrac{-2x^2+7x-11}{18(1-x)^3}, && (59)\\
H_1^{[7]\prime}(x) &= \tfrac{24x^3+52x^2-32x}{9(1-x)^4} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-189x^3-783x^2+425x+43}{81(1-x)^5}\ln x \;+\; \tfrac{-1030x^3-1899x^2+1332x+85}{243(1-x)^4}, && (60)\\
H_1^{[7]\prime\prime}(x) &= \tfrac{6x^3-62x^2+32x}{9(1-x)^5}\ln x \;+\; \tfrac{28x^3-129x^2-12x+41}{27(1-x)^4}, && (61)\\
H_2^{[7]\prime}(x) &= \tfrac{112x^2-48x}{9(1-x)^3} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{12x^3-176x^2+64x+16}{9(1-x)^4}\ln x \;+\; \tfrac{-170x^2+66x+20}{9(1-x)^3}, && (62)\\
H_2^{[7]\prime\prime}(x) &= \tfrac{12x^3-88x^2+40x}{9(1-x)^4}\ln x \;+\; \tfrac{-14x^2-54x+32}{9(1-x)^3}, && (63)\\
H_1^{[8]\prime}(x) &= \tfrac{-9x^3-46x^2-49x}{12(1-x)^4} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{81x^3+594x^2+1270x+71}{108(1-x)^5}\ln x \;+\; \tfrac{923x^3+3042x^2+6921x+1210}{648(1-x)^4}, && (64)\\
H_1^{[8]\prime\prime}(x) &= \tfrac{5x^2+19x}{3(1-x)^5}\ln x \;+\; \tfrac{7x^3-30x^2+141x+26}{18(1-x)^4}, && (65)\\
H_2^{[8]\prime}(x) &= \tfrac{-16x^2-12x}{3(1-x)^3} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{52x^2+109x+7}{6(1-x)^4}\ln x \;+\; \tfrac{95x^2+180x+61}{12(1-x)^3}, && (66)\\
H_2^{[8]\prime\prime}(x) &= \tfrac{10x^2+26x}{3(1-x)^4}\ln x \;+\; \tfrac{-x^2+30x+7}{3(1-x)^3}. && (67)
\end{aligned}
$$

Contributions to the primed coefficients can be obtained by interchanging X^{U_L} and X^{U_R} in eqs. (57) and (58). Their suppression by m_s/m_b becomes transparent only after expressing X^{U_R} in terms of other SSM parameters (cf. eq. (90) below).

Very similar substitutions need to be made for loops containing the neutralino $\tilde{\chi}^0_I$ $(I = 1,2,3,4)$ and the down squark $\tilde{d}_A$ $(A = 1,...,6)$: $m_\psi \to m_{\tilde{\chi}^0_I}$, $m_\phi \to m_{\tilde{d}_A}$, $Q_\psi \to 0$, $Q_\phi \to -1/3$, $\kappa_\psi \to 0$, $\kappa_\phi \to 4/3$ and

$$
\begin{aligned}
N\left(S_L^* B_L + \frac{m_s}{m_b} S_R^* B_R\right) &= \frac{1}{g_2^2 K_{ts}^* K_{tb}} \frac{M_W^2}{m_{\tilde{d}_A}^2} \left(Z_I^{D_L}\right)^*_{A2} \left(Z_I^{D_L}\right)_{A3} \;+\; \mathcal{O}\left(\frac{m_s^2}{M_W^2}\right),\\
N S_L^* B_R \frac{m_\psi}{m_b} &= \frac{1}{g_2^2 K_{ts}^* K_{tb}} \frac{M_W^2}{m_{\tilde{d}_A}^2} \left(Z_I^{D_L}\right)^*_{A2} \left(Z_I^{D_R}\right)_{A3} \frac{m_{\tilde{\chi}^0_I}}{m_b}. \qquad (68)
\end{aligned}
$$

One could ask whether the results obtained in section 2 for heavy Dirac fermions can be applied to the case of neutralinos, which are Majorana particles. The answer to this question is positive: for the particular set of Feynman diagrams we have considered, there is technically no difference between Dirac and Majorana fermions on internal lines. No "clashing-arrow" propagators have to be included, and no extra combinatoric factors occur.

Application of the above substitutions gives us the following neutralino contributions to the Wilson coefficients:

$$
\delta^{\tilde{\chi}^0} C_4^{(1)}(\mu_0) = \frac{1}{g_2^2 K_{ts}^* K_{tb}} \sum_{A=1}^{6}\sum_{I=1}^{4} \frac{M_W^2}{m_{\tilde{\chi}^0_I}^2} \left(Z_I^{D_L}\right)^*_{A2} \left(Z_I^{D_L}\right)_{A3} H_1^{[4]}(w_{AI}) \qquad \text{with} \quad w_{AI} = \frac{m_{\tilde{d}_A}^2}{m_{\tilde{\chi}^0_I}^2}, \quad (69)
$$

$$
\begin{aligned}
\delta^{\tilde{\chi}^0} C_{7,8}(\mu_0) &= \frac{1}{g_2^2 K_{ts}^* K_{tb}} \sum_{A=1}^{6} \sum_{I=1}^{4} \frac{M_W^2}{m_{\tilde{\chi}_I^0}^2} \times \\
&\times \left\{ \left(Z_I^{D_L}\right)^*_{A2} \left(Z_I^{D_L}\right)_{A3} \left[H_3^{[7,8]}(w_{AI}) + \frac{\alpha_s}{4\pi} \left(H_3^{[7,8]\prime}(w_{AI}) + H_3^{[7,8]\prime\prime}(w_{AI}) \ln\left(\frac{\mu_0^2}{m_{\tilde{d}_A}^2}\right)\right)\right] \right. \\
&+ \left. \frac{m_{\tilde{\chi}_I^-}}{m_b} \left(Z_I^{D_L}\right)^*_{A2} \left(Z_I^{D_R}\right)_{A3} \left[H_4^{[7,8]}(w_{AI}) + \frac{\alpha_s}{4\pi} \left(H_4^{[7,8]\prime}(w_{AI}) + H_4^{[7,8]\prime\prime}(w_{AI}) \ln\left(\frac{\mu_0^2}{m_{\tilde{d}_A}^2}\right)\right)\right] \right\}, \quad (70)
\end{aligned}
$$

where $H_3^{[8]}(x) = H_1^{[8]}(x)$, $H_4^{[8]}(x) = H_2^{[8]}(x) + \frac{1}{2}$, $H_{3,4}^{[8]\prime}(x) = H_{1,2}^{[8]\prime}(x)$, $H_{3,4}^{[8]\prime\prime}(x) = H_{1,2}^{[8]\prime\prime}(x)$, $H_3^{[7]}(x) = -\frac{1}{3} H_1^{[8]}(x)$, $H_4^{[7]}(x) = -\frac{1}{3}\left(H_2^{[8]}(x) + \frac{1}{2}\right)$ and

$$H_3^{[7]\prime}(x) = \tfrac{16x^2+28x}{9(1-x)^4} Li_2\left(1-\tfrac{1}{x}\right) + \tfrac{-108x^2-358x-38}{81(1-x)^5} \ln x + \tfrac{23x^3-765x^2-693x-77}{243(1-x)^4}, \quad (71)$$

$$H_3^{[7]\prime\prime}(x) = \tfrac{4x^2-28x}{9(1-x)^5} \ln x + \tfrac{-8x^3+42x^2-84x-22}{27(1-x)^4}, \quad (72)$$

$$H_4^{[7]\prime}(x) = \tfrac{16x^2+48x}{9(1-x)^3} Li_2\left(1-\tfrac{1}{x}\right) + \tfrac{-8x^2-68x-8}{9(1-x)^4} \ln x + \tfrac{-26x^2-54x-4}{9(1-x)^3}, \quad (73)$$

$$H_4^{[7]\prime\prime}(x) = \tfrac{8x^2-44x}{9(1-x)^4} \ln x + \tfrac{10x^2-30x-16}{9(1-x)^3}. \quad (74)$$

Contributions to the primed coefficients can be obtained by interchanging Z^{D_L} and Z^{D_R} in eqs. (69) and (70).

Finally, we turn to contributions from gluino–squark loops, i.e. from one-loop diagrams with gluinos and two-loop diagrams with both gluinos and gluons. The heavy fermion and scalar considered in section 2 are now in the adjoint and fundamental representations of $SU(3)_{colour}$, respectively. Comments concerning Majorana fermions we have made in the context of neutralinos apply here as well. The necessary substitutions in the present case are the following: $m_\psi \to m_{\tilde{g}}$, $m_\phi \to m_{\tilde{d}_A}$, $Q_\psi \to 0$, $Q_\phi \to -1/3$, $\kappa_\psi \to 3$, $\kappa_\phi \to 4/3$ and

$$
\begin{aligned}
N\left(S_L^* B_L + \frac{m_s}{m_b} S_R^* B_R\right) &= \frac{8 g_3^2}{3 g_2^2 K_{ts}^* K_{tb}} \frac{M_W^2}{m_{\tilde{d}_A}^2} \left(\Gamma^{D_L}\right)^*_{A2} \left(\Gamma^{D_L}\right)_{A3} + \mathcal{O}\left(\frac{m_s^2}{M_W^2}\right), \\
N S_L^* B_R \frac{m_\psi}{m_b} &= -\frac{8 g_3^2}{3 g_2^2 K_{ts}^* K_{tb}} \frac{M_W^2}{m_{\tilde{d}_A}^2} \left(\Gamma^{D_L}\right)^*_{A2} \left(\Gamma^{D_R}\right)_{A3} \frac{m_{\tilde{g}}}{m_b}. \quad (75)
\end{aligned}
$$

The resulting contributions to the Wilson coefficients read

$$\delta^{\tilde{g}} C_4^{(1)}(\mu_0) = \frac{8 g_3^2}{3 g_2^2 K_{ts}^* K_{tb}} \frac{M_W^2}{m_{\tilde{g}}^2} \sum_{A=1}^{6} \left(\Gamma^{D_L}\right)^*_{A2} \left(\Gamma^{D_L}\right)_{A3} H_5^{[4]}(v_A) \qquad \text{with } v_A = \frac{m_{\tilde{d}_A}^2}{m_{\tilde{g}}^2}, \quad (76)$$

$$
\begin{aligned}
\delta^{\tilde{g}} C_{7,8}(\mu_0) &= \frac{8 g_3^2}{3 g_2^2 K_{ts}^* K_{tb}} \frac{M_W^2}{m_{\tilde{g}}^2} \sum_{A=1}^{6} \times \\
&\times \left\{ \left(\Gamma^{D_L}\right)^*_{A2} \left(\Gamma^{D_L}\right)_{A3} \left[H_5^{[7,8]}(v_A) + \frac{\alpha_s}{4\pi} \left(H_5^{[7,8]\prime}(v_A) + H_5^{[7,8]\prime\prime}(v_A) \ln\left(\frac{\mu_0^2}{m_{\tilde{d}_A}^2}\right)\right)\right] \right.
\end{aligned}
$$

$$-\frac{m_{\tilde{g}}}{m_b}\left(\Gamma^{D_L}\right)^*_{A2}\left(\Gamma^{D_R}\right)_{A3}\left[H_6^{[7,8]}(v_A)+\frac{\alpha_s}{4\pi}\left(H_6^{[7,8]'}(v_A)+H_6^{[7,8]''}(v_A)\ln\left(\frac{\mu_0^2}{m^2_{\tilde{d}_A}}\right)\right)\right]\Bigg\}, \quad (77)$$

where $H_5^{[7]}(x)=-\frac{1}{3}H_1^{[8]}(x)$, $H_6^{[7]}(x)=-\frac{1}{3}\left(H_2^{[8]}(x)+\frac{1}{2}\right)$ and

$$H_5^{[4]}(x) = \frac{18x^3-27x^2+1}{24(1-x)^4}\ln x + \frac{73x^2-134x+37}{72(1-x)^3}, \quad (78)$$

$$H_5^{[7]'}(x) = \frac{17x^2+86x-15}{18(1-x)^4}Li_2\left(1-\frac{1}{x}\right) + \frac{6x^3+45x^2+66x-5}{12(1-x)^5}\ln^2 x$$
$$+ \frac{-36x^4-315x^3+1161x^2+751x+23}{162(1-x)^5}\ln x + \frac{-799x^3+1719x^2+10431x-1847}{972(1-x)^4}, \quad (79)$$

$$H_5^{[7]''}(x) = \frac{18x^3+107x^2+43x}{18(1-x)^5}\ln x + \frac{-5x^3+384x^2+609x+20}{108(1-x)^4}, \quad (80)$$

$$H_6^{[7]'}(x) = \frac{19x^2+60x-15}{9(1-x)^3}Li_2\left(1-\frac{1}{x}\right) + \frac{6x^3+36x^2+48x-5}{6(1-x)^4}\ln^2 x$$
$$+ \frac{-27x^3+106x^2+52x+1}{9(1-x)^4}\ln x + \frac{14x^2+333x-83}{18(1-x)^3}, \quad (81)$$

$$H_6^{[7]''}(x) = \frac{18x^3+80x^2+28x}{9(1-x)^4}\ln x + \frac{55x^2+69x+2}{9(1-x)^3}, \quad (82)$$

$$H_5^{[8]}(x) = \frac{9x^2-x}{16(1-x)^4}\ln x + \frac{19x^2+40x-11}{96(1-x)^3}, \quad (83)$$

$$H_5^{[8]'}(x) = \frac{45x^3-1208x^2+901x-570}{96(1-x)^4}Li_2\left(1-\frac{1}{x}\right) + \frac{-237x^3-846x^2+282x-95}{32(1-x)^5}\ln^2 x$$
$$+ \frac{2520x^4-10755x^3-10638x^2-6427x-44}{864(1-x)^5}\ln x + \frac{5359x^3-241425x^2+143253x-59251}{5184(1-x)^4}, \quad (84)$$

$$H_5^{[8]''}(x) = \frac{-747x^3-640x^2+43x}{48(1-x)^5}\ln x + \frac{-779x^3-7203x^2-93x+11}{288(1-x)^4}, \quad (85)$$

$$H_6^{[8]}(x) = \frac{9x^2-x}{8(1-x)^3}\ln x + \frac{13x-5}{8(1-x)^2}, \quad (86)$$

$$H_6^{[8]'}(x) = \frac{-359x^2+339x-204}{24(1-x)^3}Li_2\left(1-\frac{1}{x}\right) + \frac{-78x^3-333x^2+105x-34}{8(1-x)^4}\ln^2 x$$
$$+ \frac{-207x^3-1777x^2+23x-151}{48(1-x)^4}\ln x + \frac{-1667x^2+990x-379}{24(1-x)^3}, \quad (87)$$

$$H_6^{[8]''}(x) = \frac{-126x^3-133x^2+7x}{6(1-x)^4}\ln x + \frac{-553x^2+84x-35}{12(1-x)^3}. \quad (88)$$

Contributions to the primed coefficients can be obtained by interchanging Γ^{D_L} and Γ^{D_R} in eqs. (76) and (77).

Our leading-order SSM results agree with refs. [1, 2, 25]. The next-to-leading results are new, except for the chargino ones, which will be compared with those of ref. [2] in the next section.

6. Numerical size of the NLO corrections

In the present section, we shall give a numerical example of the NLO matching effect in the $B \to X_s\gamma$ branching ratio. From among many possible extensions of the SM to which our results can be applied, we choose the Supersymmetric Standard Model, because of its current

popularity. Our present results allow us to make the NLO prediction for $BR[B \to X_s\gamma]$ in the SSM only in certain regions of its parameter space. We cannot consider situations where the gluino mass is close in size to M_W or m_t, because two-loop matching diagrams with no gluons (but only gluinos) have not been calculated so far.[8] In addition, we have to assume that $\tan\beta$ is not much larger than unity. Otherwise, two-loop diagrams containing no QCD interactions at all would become numerically relevant, and the analysis would be much more involved.

For simplicity, we shall consider a scenario in which the gluino is much heavier than *all* the other SSM particles, and we shall neglect all the $1/m_{\tilde{g}}$ effects. This scenario is somewhat different from the one considered in ref. [2], where some of the other superpartners were considered heavy as well.

So long as only the gluino is heavy and decouples, QCD corrections to loops with other supersymmetric particles are given by two-loop diagrams with gluons (calculated in the previous sections) and by two-loop diagrams with strong quartic squark couplings (found in appendix C). One-loop gluino corrections to the Wilson coefficients of four-quark operators vanish in this limit. However, we need to take into account the fact that the theory with decoupled gluino is no longer supersymmetric. Consequently, the usual tree-level expressions for chargino and neutralino couplings are affected by $\mathcal{O}(\alpha_s)$ corrections, which do *not* vanish in the $m_{\tilde{g}} \to \infty$ limit.

When the gluino mass $m_{\tilde{g}}$ is much larger than masses of all the other SSM particles, one should, in principle, perform the decoupling in two steps. First, one should match the complete SSM with the effective theory built out of all the SSM particles except for the gluino. This matching should be performed at the renormalization scale $\mu_{\tilde{g}}$ that should be of the same order as $m_{\tilde{g}}$. Next, one should perform the RGE evolution of all the couplings in the obtained effective theory down to the scale μ_0 that should be of the same order as M_W or m_t. At this scale, all the remaining SSM particles with masses $\mathcal{O}(M_W, m_t)$ are decoupled, and the Wilson coefficients C_4, C_7 and C_8 are found.

The scales $\mu_{\tilde{g}}$ and μ_0 can be set equal to each other so long as $\ln \frac{\mu_{\tilde{g}}}{\mu_W}$ is not much larger than unity. This can happen even in situations where the gluino is still heavy enough, for instance when $m_{\tilde{g}} \sim 700$ GeV, and all the remaining SSM particles have masses below, say, 350 GeV. The results can then be written in a compact form, because no RGE evolution

[8] It would require including one-loop gluino corrections to the Wilson coefficients of four-quark operators, too.

occurs between $\mu_{\tilde{g}}$ and μ_0. We shall take advantage of this opportunity in our example here.

Once the gluino is decoupled, chargino and neutralino couplings $X^{U_{L,R}}$, $Z^{D_{L,R}}$ are related to the other parameters of the model as follows (cf. eq. (2.12) of ref. [25]):

$$X_I^{U_L} = g_2 \left[-a_g V_{I1}^* \Gamma^{U_L} + a_Y V_{I2}^* \Gamma^{U_R} \frac{M_U}{\sqrt{2} M_W \sin\beta} \right] K, \tag{89}$$

$$X_I^{U_R} = g_2 a_Y U_{I2} \Gamma^{U_L} K \frac{M_D}{\sqrt{2} M_W \cos\beta}, \tag{90}$$

$$Z_I^{D_L} = -\frac{g_2}{\sqrt{2}} \left[a_g \left(-N_{I2}^* + \frac{1}{3} \tan\theta N_{I1}^* \right) \Gamma^{D_L} + a_Y N_{I3}^* \Gamma^{D_R} \frac{M_D}{M_W \cos\beta} \right], \tag{91}$$

$$Z_I^{D_R} = -\frac{g_2}{\sqrt{2}} \left[a_g \frac{2}{3} \tan\theta N_{I1} \Gamma^{D_R} + a_Y N_{I3} \Gamma^{D_L} \frac{M_D}{M_W \cos\beta} \right], \tag{92}$$

where

$$a_g = 1 + \frac{\alpha_s(\mu_0)}{\pi} \left(\ln \frac{m_{\tilde{g}}}{\mu_0} - \frac{7}{12} \right) \qquad \text{and} \qquad a_Y = 1 - \frac{\alpha_s(\mu_0)}{\pi} \left(\ln \frac{m_{\tilde{g}}}{\mu_0} - \frac{1}{4} \right). \tag{93}$$

In the above equations, all the parameters (including the matrices $X^{U_{L,R}}$ and $Z^{D_{L,R}}$) are assumed to be $\overline{MS}$-renormalized in dimensional *regularization*, in the effective theory containing no gluino. Setting a_g and a_Y to unity, we would obtain relations for parameters of the full SSM, $\overline{MS}$-renormalized in dimensional *reduction*. Explicit values of a_g and a_Y have been obtained by performing simple one-loop matching between these two theories.

One might wonder whether the appearance of a_g and a_Y is the only effect of gluino decoupling. Potentially, interactions different from those of the SSM could be generated at one loop. A more detailed examination of the relevant one-loop diagrams leads to the conclusion that other effects indeed occur, but they are never relevant to the NLO QCD corrections to $BR[B \to X_s \gamma]$.

Once a_Y and a_g from eq. (93) are included, we are able to verify that our results for the NLO corrections to chargino loops (eqs. (58)–(67), (106) and (107)) agree with those of ref. [2], so long as the gluino is assumed to be much heavier than all the other SSM particles. When performing the comparison, one needs to take into account that our results are expressed in terms of $\overline{MS}$-renormalized masses, while the on-shell squark masses were used in ref. [2].

The last worry one could have in the context of the SSM is whether two-loop effects involving no couplings proportional to α_s, but only to the large Yukawa coupling Y_t, could be of a numerical importance similar to the NLO QCD corrections. This can potentially

happen even in the small $\tan\beta$ regime. No definite answer to this question can be given until the SM calculation of ref. [22] is generalized to the SSM case.[9] However, the main purpose of the present section is only to demonstrate that $\mathcal{O}(\alpha_s)$ effects can be sizable by themselves, independently of the magnitude of two-loop higgson/higgsino contributions.

Having made all the necessary assumptions, we are now ready to collect the results from section 5 and appendix C to test the size of $\mathcal{O}(\alpha_s)$ corrections to $BR[B \to X_s\gamma]$ for some particular values of the SSM parameters. A relatively simple set of SSM parameters, which is allowed by all the experimental constraints (including $B \to X_s\gamma$), can be chosen as follows:

$$\begin{aligned}
\tan\beta &= 3, & m_{\tilde{W}}/\mu = -2,\\
m_{h^\pm} &= 100 \text{ GeV}, & m_{gluino} = 700 \text{ GeV},\\
(M_{\tilde{\chi}^\pm})_{11} &= 140 \text{ GeV} \quad \text{(the lighter chargino mass)}, &\\
(M^2_{\tilde{u}})_{AB} &= (\delta_{AB} - \delta_{A6}\delta_{B6}) \times (350 \text{ GeV})^2 + \delta_{A6}\delta_{B6} \times (110 \text{ GeV})^2, &\\
\Gamma^U_{AB} &\simeq \delta_{AB} + (\cos 25^o - 1)(\delta_{A3}\delta_{B3} + \delta_{A6}\delta_{B6}) + \sin 25^o(\delta_{A6}\delta_{B3} - \delta_{A3}\delta_{B6}), &\\
\Gamma^D &\simeq \quad \text{(flavour-diagonal matrix)}. &
\end{aligned}$$

The first three parameters determine the chargino masses and mixing angles. Down-squark masses, neutralino masses and mixing angles are irrelevant here, because we have assumed approximate vanishing of flavour violation in the down squark mass matrix. Consequently, all the neutralino contributions to the Wilson coefficients $C_{4,7,8}$ vanish. As far as the relevant SM parameters are concerned, we take the same values as in ref. [14], i.e. $m_{t,pole} = 175$ GeV

$\delta^{SM} C_7^{(0)}(M_W) = -0.197$	$\delta^{SM} C_8^{(0)}(M_W) = -0.098$	
$\delta^{H^-} C_7^{(0)}(M_W) = -0.279$	$\delta^{H^-} C_8^{(0)}(M_W) = -0.211$	
$\delta^{\tilde{\chi}^-} C_7^{(0)}(M_W) = 0.272$	$\delta^{\tilde{\chi}^-} C_8^{(0)}(M_W) = 0.148$	
$\delta^{SM} C_7^{(1)}(M_W) = -2.49$	$\delta^{SM} C_8^{(1)}(M_W) = -2.22$	$\delta^{SM} C_4^{(1)}(M_W) = -0.42$
$\delta^{H^-} C_7^{(1)}(M_W) = 5.24$	$\delta^{H^-} C_8^{(1)}(M_W) = 2.85$	$\delta^{H^-} C_4^{(1)}(M_W) = 0.02$
$\delta^{\tilde{\chi}^-} C_7^{(1)}(M_W) = -0.24$	$\delta^{\tilde{\chi}^-} C_8^{(1)}(M_W) = 0.18$	$\delta^{\tilde{\chi}^-} C_4^{(1)}(M_W) = 0.08$
$\delta_q^{\tilde{\chi}^-} C_7^{(1)}(M_W) = -2.22$	$\delta_q^{\tilde{\chi}^-} C_8^{(1)}(M_W) = -1.49$	

Table 1. Numerical contributions to the Wilson coefficients in the considered example

[9] The smallness of the net effect in the SM is partly due to accidental cancellations, which may not take place in the SSM.

and $\alpha_s(M_Z) = 0.118$. For the Wolfenstein parameters, we choose $\lambda = 0.22,\ \ A = 0.83,\ \ \rho = 0$ and $\eta = 0.3$. Such ρ and η are roughly in the middle of the allowed region in the SM. They are acceptable also in the SSM, for the parameters we have specified above.

The contributions to the Wilson coefficients obtained at $\mu_0 = M_W$ are given in table 1. The $\mathcal{O}(\alpha_s \ln m_{\tilde{g}}^2/\mu_0^2)$ terms in chargino and neutralino couplings have been included both in the leading and next-to-leading contributions to the Wilson coefficients, as should be done in the SSM with decoupled gluino.

Having found the SSM matching conditions numerically, we evaluate $BR[B \to X_s\gamma]$ according to the formulae of ref. [14] and using the same input parameters. However, we choose a different photon energy cut-off $\delta = 0.90$ [19], change $\alpha_{em}(m_b)$ to $\alpha_{em}^{on\ shell} \simeq \frac{1}{137}$ in the overall normalization [18], and include the $1/m_c$ corrections [21]. The final result for $BR[B \to X_s\gamma]$ is

$$BR[B \to X_s\gamma] = 3.15 \times 10^{-4}, \tag{94}$$

which is equal to the central value of the CLEO measurement quoted in the introduction. On the other hand, if we did not include the two-loop gluonic SUSY contributions (i.e. if we set $\delta^{H^-} C_{7,8}^{(1)}(M_W)$ and $\delta^{\tilde{\chi}^-} C_{7,8}^{(1)}(M_W)$ to zero), we would obtain $BR[B \to X_s\gamma] = 3.69 \times 10^{-4}$. The difference between the two results is close in size to the present experimental uncertainty. Such a size of the two-loop SUSY effect is rather generic for light supersymmetric particles, for which the leading SUSY contributions to $B \to X_s\gamma$ are of the same order as the SM one. Thus, QCD corrections to superpartner loops are expected to be important when making a meaningful comparison with experiment in such cases.

Obviously, satisfying the experimental $B \to X_s\gamma$ bound for light superpartners requires a certain adjustment of the SSM parameters. In our case, the adjusted parameter was the lighter chargino mass. The experimental $B \to X_s\gamma$ bound would be satisfied within 1σ for this mass ranging from 110 to 175 GeV, i.e. no real fine-tuning was necessary.

For larger $\tan\beta$, SUSY contributions to $B \to X_s\gamma$ can be much larger than the SM one. Then, the NLO SUSY effects are much more important than in the example presented above. However, satisfying the experimental $B \to X_s\gamma$ constraint requires real fine-tuning then.

Our results for the two-loop SUSY contributions to $B \to X_s\gamma$ could be used in scans over the SSM parameter space. The allowed regions in this space would change their position for light superpartners. However, our feeling is that making such scans at present would be somewhat premature. The NLO SUSY contributions to $B \to X_s\gamma$ can become qualitatively

important only when either light superpartners are discovered or their existence is almost excluded by data combined from many experiments. One of these options will certainly be realized when the LHC starts collecting data. At that time, one might appreciate the usefulness of the long analytic formulae contained in the present paper.

7. Summary

We have calculated matching conditions for operators mediating the $b \to s\gamma$ and $b \to s$ *gluon* transitions in a large class of extensions of the Standard Model. Both the leading one-loop diagrams and the gluonic corrections to them have been included. Taking the Supersymmetric Standard Model as an example, we have checked that QCD corrections to new physics contributions can be close in size to the present experimental uncertainty, even when the SSM parameters are not really fine-tuned. A similar situation is expected to occur in other theories of new physics containing exotic particles at the electroweak scale.

The main purpose of our paper was to present complete analytic formulae for the NLO Wilson coefficients in a possibly generic extension of the SM. From these results, we could reproduce the known matching conditions in the Standard Model, the Two-Higgs Doublet Model, and for the chargino contributions in the SSM. New results were obtained for the left–right-symmetric model, as well as for neutralino and gluino contributions in the SSM. Our SSM results form a major contribution to the complete SSM calculation. They become complete by themselves in the heavy-gluino limit and for $\tan\beta$ of order unity.

Acknowledgements

C.B and J.U. thank F. Krauss, K. Schubert and G. Soff for helpful discussions. M.M. thanks Andrzej Buras and Paolo Gambino for useful advice. This work has been supported in part by the German Bundesministerium für Bildung und Forschung under contracts 06 DD 823 (J.U.) and 06 TM 874 (M.M). M.M. has been supported in part by the DFG project Li 519/2-2, as well as by the Polish Committee for Scientific Research under grant 2 P03B 014 14, 1998-2000.

Appendix A

In this appendix, we summarize several useful identities involving colour generators and the Clebsch–Gordan coefficients C_{ijk}. Thanks to them, all the colour factors in section 2 could have been expressed only in terms of κ_ψ, κ_ϕ, and ξ (cf. eqs. (8) and (20)):

$$\begin{aligned}
T^a_{(\phi)ij} T^a_{(\psi)kl} C_{jlm} &= \frac{1}{2}\left(\frac{4}{3} - \kappa_\phi - \kappa_\psi\right) C_{ikm}, && (95)\\
T^a_{(\phi)ij} C_{jkl} T^a_{lm} &= \frac{1}{2}\left(\frac{4}{3} + \kappa_\phi - \kappa_\psi\right) C_{ikm}, && (96)\\
T^a_{(\psi)kj} C_{ijl} T^a_{lm} &= \frac{1}{2}\left(\frac{4}{3} + \kappa_\psi - \kappa_\phi\right) C_{ikm}, && (97)\\
C^*_{ijn} T^a_{(\phi)il} C_{ljm} &= \frac{3}{8}\xi\left(\frac{4}{3} + \kappa_\phi - \kappa_\psi\right) T^a_{nm}, && (98)\\
C^*_{jin} T^a_{(\psi)il} C_{jlm} &= \frac{3}{8}\xi\left(\frac{4}{3} + \kappa_\psi - \kappa_\phi\right) T^a_{nm}, && (99)\\
C^*_{ijn}\left(T^a_{(\phi)} T^b_{(\phi)}\right)_{il} C_{ljm} &= \frac{\xi}{8}\left[\kappa_\phi \delta_{ab}\delta_{nm} + \frac{3i}{2}\left(\frac{4}{3} + \kappa_\phi - \kappa_\psi\right) f_{abc} T^c_{nm} + \frac{3}{5}\eta_\phi d_{abc} T^c_{nm}\right], && (100)\\
C^*_{jin}\left(T^a_{(\psi)} T^b_{(\psi)}\right)_{il} C_{jlm} &= \frac{\xi}{8}\left[\kappa_\psi \delta_{ab}\delta_{nm} + \frac{3i}{2}\left(\frac{4}{3} + \kappa_\psi - \kappa_\phi\right) f_{abc} T^c_{nm} + \frac{3}{5}\eta_\psi d_{abc} T^c_{nm}\right], && (101)
\end{aligned}$$

where

$$\begin{aligned}
\eta_\phi &= 3(\kappa_\phi - \kappa_\psi)^2 + \frac{3}{2}\kappa_\phi - \frac{7}{2}\kappa_\psi - \frac{2}{3}, && (102)\\
\eta_\psi &= 3(\kappa_\psi - \kappa_\phi)^2 + \frac{3}{2}\kappa_\psi - \frac{7}{2}\kappa_\phi - \frac{2}{3}. && (103)
\end{aligned}$$

All these identities can be derived from the basic constraint given in eq. (6).

For completeness, let us quote the standard identities for the fundamental representation, too:

$$T^a T^b = \frac{i}{2} f_{abc} T^c + \frac{1}{2} d_{abc} T^c + \frac{1}{6}\delta_{ab} 1, \tag{104}$$

$$f_{abc} f_{abd} = 3\delta_{cd}, \qquad d_{abc} d_{abd} = \frac{5}{3}\delta_{cd}, \qquad d_{aab} = 0. \tag{105}$$

Appendix B

Here, we present explicit formulae for the functions $f_i(x)$, $g_i(x)$, $h_i(x)$, $j_i(x)$ and $k_i(x)$ introduced in sections 2 and 3. All these expressions are available in Mathematica format via anonymous ftp from ftp://feynman.t30.physik.tu-muenchen.de/pub/preprints/tum-hep-321-98.functions.m . They read:

$$f_1 = \frac{x+2}{8(1-x)^2} \ln x + \frac{3}{8(1-x)},$$

$$f_2 = \frac{x^3+3x-2}{6(1-x)^4} \ln x + \frac{2x^2+11x-7}{18(1-x)^3},$$

$$
\begin{aligned}
g_1 &= \tfrac{-x}{2(1-x)^4}\ln x \;+\; \tfrac{x^2-5x-2}{12(1-x)^3}, \\
g_2 &= \tfrac{1}{(1-x)^3}\ln x \;+\; \tfrac{-x+3}{2(1-x)^2}, \\
h_1 &= \tfrac{2x-3}{3(1-x)^2} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{6x^3+63x^2+49x-6}{12(1-x)^5}\ln^2 x \;+\; \tfrac{-13x^4-65x^3-369x^2-191x+62}{36(1-x)^5}\ln x \;+\; \tfrac{-20x^3+59x^2-242x+11}{12(1-x)^4}, \\
h_2 &= \tfrac{-x+3}{3(1-x)^2} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-6x^3+9x^2-41x+6}{12(1-x)^5}\ln^2 x \;+\; \tfrac{11x^4-32x^3+222x^2+82x-31}{18(1-x)^5}\ln x \;+\; \tfrac{2x^3+11x^2+172x-17}{12(1-x)^4}, \\
h_3 &= \tfrac{4x^3+4x^2+112x+12}{9(1-x)^4} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-6x^3-39x^2+71x+6}{9(1-x)^5}\ln^2 x \;+\; \tfrac{11x^3-150x^2-39x+62}{27(1-x)^4}\ln x \;+\; \tfrac{-5x^2+76x+481}{81(1-x)^3}, \\
h_4 &= \tfrac{-5x}{6(1-x)^2} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{51x^3+60x^2+x}{12(1-x)^5}\ln^2 x \;+\; \tfrac{-11x^4-163x^3-153x^2+35x+4}{18(1-x)^5}\ln x \;+\; \tfrac{19x^3-163x^2-59x+11}{12(1-x)^4}, \\
h_5 &= \tfrac{7x}{6(1-x)^2} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-3x^3-30x^2+x}{12(1-x)^5}\ln^2 x \;+\; \tfrac{2x^4+112x^3+177x^2-35x-4}{18(1-x)^5}\ln x \;+\; \tfrac{-4x^3+161x^2+16x-5}{12(1-x)^4}, \\
h_6 &= \tfrac{-14x^3+112x^2+34x}{9(1-x)^4} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-39x^3+48x^2+23x}{9(1-x)^5}\ln^2 x \;+\; \tfrac{-34x^3-168x^2+78x+8}{27(1-x)^4}\ln x \;+\; \tfrac{-113x^2+643x+22}{81(1-x)^3}, \\
h_7 &= \tfrac{4x-10}{3(1-x)^2} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-6x^2-51x-28}{6(1-x)^4}\ln^2 x \;+\; \tfrac{6x^3-8x^2+77x-3}{3(1-x)^4}\ln x \;+\; \tfrac{-x^2+3x+142}{6(1-x)^3}, \\
h_8 &= \tfrac{-2x+8}{3(1-x)^2} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{6x^2-3x+20}{6(1-x)^4}\ln^2 x \;+\; \tfrac{-6x^3+19x^2-85x+9}{3(1-x)^4}\ln x \;+\; \tfrac{23x^2-90x-59}{6(1-x)^3}, \\
h_9 &= \tfrac{-8x^2-8x-176}{9(1-x)^3} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{12x^2+54x-112}{9(1-x)^4}\ln^2 x \;+\; \tfrac{-24x^2+124x-24}{9(1-x)^3}\ln x \;+\; \tfrac{34x-104}{9(1-x)^2}, \\
h_{10} &= \tfrac{-5x-1}{3(1-x)^2} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-39x^2-39x-7}{6(1-x)^4}\ln^2 x \;+\; \tfrac{3x^3+40x^2+38x-9}{3(1-x)^4}\ln x \;+\; \tfrac{-25x^2+147x+22}{6(1-x)^3}, \\
h_{11} &= \tfrac{7x-1}{3(1-x)^2} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{9x^2+9x+5}{6(1-x)^4}\ln^2 x \;+\; \tfrac{-3x^3-29x^2-40x+9}{3(1-x)^4}\ln x \;+\; \tfrac{5x^2-138x+7}{6(1-x)^3}, \\
h_{12} &= \tfrac{28x^2-176x-44}{9(1-x)^3} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{54x^2-66x-34}{9(1-x)^4}\ln^2 x \;+\; \tfrac{12x^2+100x-36}{9(1-x)^3}\ln x \;+\; \tfrac{22x-92}{9(1-x)^2}, \\
h_{13} &= \tfrac{9x-6}{16(1-x)^2} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{45x^2+42x-6}{32(1-x)^4}\ln^2 x \;+\; \tfrac{-3x^3-90x^2-69x+18}{32(1-x)^4}\ln x \;+\; \tfrac{39x^2-183x}{32(1-x)^3}, \\
h_{14} &= \tfrac{9x-6}{16(1-x)^2} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-3x^2+36x-6}{32(1-x)^4}\ln^2 x \;+\; \tfrac{3x^3-45x^2-30x+9}{16(1-x)^4}\ln x \;+\; \tfrac{3x^2-72x+6}{16(1-x)^3}, \\
h_{15} &= \tfrac{-9x+6}{8(1-x)^2} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-21x^2-39x+6}{16(1-x)^4}\ln^2 x \;+\; \tfrac{-3x^3+180x^2+129x-36}{32(1-x)^4}\ln x \;+\; \tfrac{-45x^2+327x-12}{32(1-x)^3}, \\
h_{16} &= \tfrac{-172x^2+211x-27}{96(1-x)^3} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{531x^3+639x^2+649x-27}{192(1-x)^5}\ln^2 x \\
&\qquad + \tfrac{-185x^4-1996x^3-1755x^2-898x+226}{288(1-x)^5}\ln x \;+\; \tfrac{-509x^3+65x^2-2699x+71}{192(1-x)^4}, \\
h_{17} &= \tfrac{140x^2-179x+27}{96(1-x)^3} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-147x^3+177x^2-569x+27}{192(1-x)^5}\ln^2 x \\
&\qquad + \tfrac{149x^4+1072x^3+2247x^2+790x-226}{288(1-x)^5}\ln x \;+\; \tfrac{485x^3+479x^2+1795x-71}{192(1-x)^4}, \\
h_{18} &= \tfrac{68x^3+167x^2+314x+75}{72(1-x)^4} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-171x^3+153x^2+455x+75}{144(1-x)^5}\ln^2 x \\
&\qquad + \tfrac{-65x^3-381x^2+318x+10}{216(1-x)^4}\ln x \;+\; \tfrac{1513x^2+2782x+1621}{1296(1-x)^3}, \\
h_{19} &= \tfrac{-9}{8(1-x)} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-33x-21}{16(1-x)^3}\ln^2 x \;+\; \tfrac{-3x^2+45x+6}{8(1-x)^3}\ln x \;+\; \tfrac{-3x+15}{2(1-x)^2}, \\
h_{20} &= \tfrac{-9}{8(1-x)} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{-3x-15}{16(1-x)^3}\ln^2 x \;+\; \tfrac{-3x^2+45x}{8(1-x)^3}\ln x \;+\; \tfrac{9x+33}{8(1-x)^2}, \\
h_{21} &= \tfrac{9}{4(1-x)} Li_2\left(1-\tfrac{1}{x}\right) \;+\; \tfrac{9x+9}{4(1-x)^3}\ln^2 x \;+\; \tfrac{3x^2-45x-3}{4(1-x)^3}\ln x \;+\; \tfrac{3x-93}{8(1-x)^2},
\end{aligned}
$$

$$h_{22} = \frac{59x-68}{24(1-x)^2} Li_2\left(1-\frac{1}{x}\right) + \frac{-39x^2-84x-47}{12(1-x)^4}\ln^2 x + \frac{63x^3+379x^2+701x+9}{48(1-x)^4}\ln x + \frac{71x^2+54x+451}{24(1-x)^3},$$

$$h_{23} = \frac{-43x+52}{24(1-x)^2} Li_2\left(1-\frac{1}{x}\right) + \frac{9x^2+37}{12(1-x)^4}\ln^2 x + \frac{-63x^3-203x^2-781x+39}{48(1-x)^4}\ln x + \frac{-67x^2-210x-227}{24(1-x)^3},$$

$$h_{24} = \frac{-7x^2-49x-112}{18(1-x)^3} Li_2\left(1-\frac{1}{x}\right) + \frac{6x^2-6x-46}{9(1-x)^4}\ln^2 x + \frac{3x^2+70x-39}{36(1-x)^3}\ln x + \frac{-23x-95}{36(1-x)^2},$$

$$j_1 = \frac{-2x^2+x}{(1-x)^4}\ln x + \frac{-5x^2-5x+4}{6(1-x)^3},$$

$$j_2 = \frac{-x^2}{(1-x)^4}\ln x + \frac{-2x^2-5x+1}{6(1-x)^3},$$

$$j_3 = \frac{x^2}{(1-x)^3}\ln x + \frac{3x-1}{2(1-x)^2},$$

$$j_4 = \frac{-4x}{(1-x)^3}\ln x + \frac{-2x-2}{(1-x)^2},$$

$$j_5 = \frac{2x}{(1-x)^2}\ln x + \frac{2}{(1-x)},$$

$$\begin{aligned} k_1 = {} & \frac{24x^3+144x^2-80x}{3(1-x)^4} Li_2\left(1-\frac{1}{x}\right) + \frac{32x^3+120x^2-56x}{3(1-x)^5}\ln^2 x \\ & + \frac{60x^4-1740x^3-84x^2+796x-184}{27(1-x)^5}\ln x + \frac{-1538x^3-5652x^2+4806x-1072}{81(1-x)^4}, \end{aligned}$$

$$\begin{aligned} k_2 = {} & \frac{32x^3+72x^2-16x}{3(1-x)^4} Li_2\left(1-\frac{1}{x}\right) + \frac{112x^2-16x}{3(1-x)^5}\ln^2 x \\ & + \frac{96x^4-1272x^3-48x^2+40x+32}{27(1-x)^5}\ln x + \frac{-2024x^3-954x^2-540x+62}{81(1-x)^4}, \end{aligned}$$

$$k_3 = \frac{-56x^2}{3(1-x)^3} Li_2\left(1-\frac{1}{x}\right) + \frac{-8x^3-64x^2}{3(1-x)^4}\ln^2 x + \frac{48x^3+80x^2-32x}{3(1-x)^4}\ln x + \frac{110x^2-12x-2}{3(1-x)^3},$$

$$k_4 = \frac{64x^2+192x}{3(1-x)^3} Li_2\left(1-\frac{1}{x}\right) + \frac{64x^2+224x}{3(1-x)^4}\ln^2 x + \frac{16x^3-368x^2-64x+32}{3(1-x)^4}\ln x + \frac{-136x^2-256x+8}{3(1-x)^3},$$

$$k_5 = \frac{64x^2+192x}{3(1-x)^3} Li_2\left(1-\frac{1}{x}\right) + \frac{64x^2+224x}{3(1-x)^4}\ln^2 x + \frac{16x^3-400x^2}{3(1-x)^4}\ln x + \frac{-56x^2-64x-8}{(1-x)^3},$$

$$k_6 = \frac{-64x}{3(1-x)^2} Li_2\left(1-\frac{1}{x}\right) + \frac{-16x^2-80x}{3(1-x)^3}\ln^2 x + \frac{88x^2+40x}{3(1-x)^3}\ln x + \frac{120x+8}{3(1-x)^2},$$

$$\begin{aligned} k_7 = {} & \frac{15x^3+72x^2-35x}{3(1-x)^4} Li_2\left(1-\frac{1}{x}\right) + \frac{68x^3+66x^2-38x}{3(1-x)^5}\ln^2 x \\ & + \frac{-150x^4-3534x^3+561x^2+971x-152}{54(1-x)^5}\ln x + \frac{-3857x^3-21960x^2+12501x-508}{324(1-x)^4}, \end{aligned}$$

$$k_8 = \frac{64x^2+48x}{3(1-x)^3} Li_2\left(1-\frac{1}{x}\right) + \frac{136x^2+152x}{3(1-x)^4}\ln^2 x + \frac{-20x^3-404x^2+26x+14}{3(1-x)^4}\ln x + \frac{-127x^2-220x-37}{3(1-x)^3}.$$

Appendix C

This appendix is devoted to presenting the SSM matching contributions originating from the quartic squark vertex proportional to the strong coupling constant α_s:

$$\begin{aligned} \delta_q^{\tilde{\chi}^-} C_7^{(1)}(\mu_0) = {} & \frac{1}{g_2^2 K_{ts}^* K_{tb}} \sum_{A,B,C=1}^{6} \sum_{I=1}^{2} \frac{M_W^2}{m_{\tilde{\chi}_I^-}^4} P^U_{AB} m^2_{\tilde{u}_B} P^U_{BC} \left(\ln\frac{m^2_{\tilde{u}_B}}{\mu_0^2} - 1\right) \times \\ & \times \left\{ \left(X_I^{U_L}\right)^*_{A2} \left(X_I^{U_L}\right)_{C3} \left[-q_1(z_{AI}, z_{CI}) + \frac{2}{3} q_2(z_{AI}, z_{CI})\right] \right. \\ & \left. + \frac{m_{\tilde{\chi}_I^-}}{m_b} \left(X_I^{U_L}\right)^*_{A2} \left(X_I^{U_R}\right)_{C3} \left[-q_3(z_{AI}, z_{CI}) + \frac{2}{3} q_4(z_{AI}, z_{CI})\right] \right\}, \end{aligned} \qquad (106)$$

$$\delta_q^{\tilde{\chi}^-} C_8^{(1)}(\mu_0) = \frac{1}{g_2^2 K_{ts}^* K_{tb}} \sum_{A,B,C=1}^{6} \sum_{I=1}^{2} \frac{M_W^2}{m_{\tilde{\chi}_I^-}^4} P^U_{AB} m_{\tilde{u}_B}^2 P^U_{BC} \left(\ln \frac{m_{\tilde{u}_B}^2}{\mu_0^2} - 1 \right) \times$$
$$\times \left\{ \left(X_I^{U_L}\right)^*_{A2} \left(X_I^{U_L}\right)_{C3} q_2(z_{AI}, z_{CI}) + \frac{m_{\tilde{\chi}_I^-}}{m_b} \left(X_I^{U_L}\right)^*_{A2} \left(X_I^{U_R}\right)_{C3} q_4(z_{AI}, z_{CI}) \right\}, \quad (107)$$

$$-3\delta_q^{\tilde{\chi}^0} C_7^{(1)}(\mu_0) = \delta_q^{\tilde{\chi}^0} C_8^{(1)}(\mu_0)$$
$$= \frac{1}{g_2^2 K_{ts}^* K_{tb}} \sum_{A,B,C=1}^{6} \sum_{I=1}^{4} \frac{M_W^2}{m_{\tilde{\chi}_I^0}^4} P^D_{AB} m_{\tilde{d}_B}^2 P^D_{BC} \left(\ln \frac{m_{\tilde{d}_B}^2}{\mu_0^2} - 1 \right) \times$$
$$\times \left\{ \left(Z_I^{D_L}\right)^*_{A2} \left(Z_I^{D_L}\right)_{C3} q_2(w_{AI}, w_{CI}) + \frac{m_{\tilde{\chi}_I^0}}{m_b} \left(Z_I^{D_L}\right)^*_{A2} \left(Z_I^{D_R}\right)_{C3} q_4(w_{AI}, w_{CI}) \right\}, \quad (108)$$

$$\delta_q^{\tilde{g}} C_7^{(1)}(\mu_0) = -\frac{8g_3^2}{9g_2^2 K_{ts}^* K_{tb}} \frac{M_W^2}{m_{\tilde{g}}^4} \sum_{A,B,C=1}^{6} P^D_{AB} m_{\tilde{d}_B}^2 P^D_{BC} \left(\ln \frac{m_{\tilde{d}_B}^2}{\mu_0^2} - 1 \right) \times$$
$$\times \left\{ \left(\Gamma^{D_L}\right)^*_{A2} \left(\Gamma^{D_L}\right)_{C3} q_2(v_A, v_C) - \frac{m_{\tilde{g}}}{m_b} \left(\Gamma^{D_L}\right)^*_{A2} \left(\Gamma^{D_R}\right)_{C3} q_4(v_A, v_C) \right\}, \quad (109)$$

$$\delta_q^{\tilde{g}} C_8^{(1)}(\mu_0) = \frac{3g_3^2}{g_2^2 K_{ts}^* K_{tb}} \frac{M_W^2}{m_{\tilde{g}}^4} \sum_{A,B,C=1}^{6} P^D_{AB} m_{\tilde{d}_B}^2 P^D_{BC} \left(\ln \frac{m_{\tilde{d}_B}^2}{\mu_0^2} - 1 \right) \times$$
$$\times \left\{ \left(\Gamma^{D_L}\right)^*_{A2} \left(\Gamma^{D_L}\right)_{C3} \left[q_1(v_A, v_C) - \frac{1}{9} q_2(v_A, v_C) \right] \right.$$
$$\left. - \frac{m_{\tilde{g}}}{m_b} \left(\Gamma^{D_L}\right)^*_{A2} \left(\Gamma^{D_R}\right)_{C3} \left[q_3(v_A, v_C) - \frac{1}{9} q_4(v_A, v_C) \right] \right\}, \quad (110)$$

where $P^U = \Gamma^{U_L}\Gamma^{U_L\dagger} - \Gamma^{U_R}\Gamma^{U_R\dagger}$ and $P^D = \Gamma^{D_L}\Gamma^{D_L\dagger} - \Gamma^{D_R}\Gamma^{D_R\dagger}$. The mass ratios are denoted as before: $z_{AI} = m_{\tilde{u}_A}^2/m_{\tilde{\chi}_I^-}^2$, $w_{AI} = m_{\tilde{d}_A}^2/m_{\tilde{\chi}_I^0}^2$ and $v_A = m_{\tilde{d}_A}^2/m_{\tilde{g}}^2$.

The explicit expressions for the functions $q_i(x,y)$ read

$$q_1(x,y) = \frac{2}{3(x-y)} \left[\frac{x^2 \ln x}{(1-x)^4} - \frac{y^2 \ln y}{(1-y)^4} \right] + \frac{2x^2y^2+5x^2y+5xy^2-x^2-y^2-22xy+5x+5y+2}{9(1-x)^3(1-y)^3}, \quad (111)$$

$$q_2(x,y) = \frac{2}{3(x-y)} \left[\frac{x \ln x}{(1-x)^4} - \frac{y \ln y}{(1-y)^4} \right] + \frac{-x^2y^2+5x^2y+5xy^2+2x^2+2y^2-10xy-7x-7y+11}{9(1-x)^3(1-y)^3}, \quad (112)$$

$$q_3(x,y) = \frac{4}{3(x-y)} \left[\frac{x^2 \ln x}{(1-x)^3} - \frac{y^2 \ln y}{(1-y)^3} \right] + \frac{-6xy+2x+2y+2}{3(1-x)^2(1-y)^2}, \quad (113)$$

$$q_4(x,y) = \frac{4}{3(x-y)} \left[\frac{x \ln x}{(1-x)^3} - \frac{y \ln y}{(1-y)^3} \right] + \frac{-2xy-2x-2y+6}{3(1-x)^2(1-y)^2}. \quad (114)$$

References

[1] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. **B353** (1991) 591.

[2] M. Ciuchini, G. Degrassi, P. Gambino and G.F. Giudice, Nucl. Phys. **B527** (1998) 21, Nucl. Phys. **B534** (1998) 3.

[3] B. Grinstein, R. Springer and M.B. Wise, Nucl. Phys. **B339** (1990) 269.

[4] F. Borzumati and C. Greub, Phys. Rev. **D58** (1998) 074004.

[5] P. Cho and M. Misiak, Phys. Rev. **D49** (1994) 5894.

[6] K. Fujikawa and A. Yamada, Phys. Rev. **D49** (1994) 5890.

[7] G.M. Asatrian and A. Ioannissian, Phys. Rev. **D54** (1996) 5642.

[8] J. Hewett, "Top ten models constrained by $b \to s\gamma$", hep-ph/9406302, in Proc. "Spin structure in high energy processes", p. 463, Stanford, 1993.

[9] S. Glenn et al. (CLEO Collaboration), preprint CLEO-CONF 98-17, ICHEP98 1011; J. Alexander, Plenary talk, to appear in Proc. ICHEP98, Vancouver, 1998.

[10] R. Barate *et al.* (ALEPH Collaboration), Phys. Lett. **B429** (1998) 169.

[11] A. Ali and C. Greub, Z. Phys. **C49** (1991) 431, Phys. Lett. **B259** (1991) 182.

[12] K. Adel and Y.P. Yao, Phys. Rev. **D49** (1994) 4945.

[13] C. Greub, T. Hurth and D. Wyler, Phys. Lett. **B380** (1996) 385, Phys. Rev. **D54** (1996) 3350.

[14] K. Chetyrkin, M. Misiak and M. Münz, Phys. Lett. **B400** (1997) 206.

[15] N. Pott, Phys. Rev. **D54** (1996) 938.

[16] C. Greub and T. Hurth, Phys. Rev. **D56** (1997) 2934.

[17] A.J. Buras, A. Kwiatkowski and N. Pott, Phys. Lett. **B414** (1997) 157, Nucl. Phys. **B517** (1998) 353.

[18] A. Czarnecki and W.J. Marciano, Phys. Rev. Lett. **81** (1998) 277.

[19] A.L. Kagan and M. Neubert, Eur. Phys. J. **C7** (1999) 5;
M. Neubert, hep-ph/9809377, to appear in Proc. ICHEP98, Vancouver, 1998.

[20] C. Greub and T. Hurth, hep-ph/9809468.

[21] M.B. Voloshin, Phys. Lett. **B397** (1997) 275;
A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, Phys. Lett. **B402** (1997) 167;
Z. Ligeti, L. Randall and M.B. Wise, Phys. Lett. **B402** (1997) 178;
A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei, Phys. Rev. **D56** (1997) 3151;
G. Buchalla, G. Isidori and S.J. Rey, Nucl. Phys. **B511** (1998) 594;

[22] A. Strumia, Nucl. Phys. **B532** (1998) 28.

[23] K. Chetyrkin, M. Misiak and M. Münz, Nucl. Phys. **B518** (1998) 473.

[24] C. Bobeth, M. Misiak and J. Urban, preprint TUM-HEP-329/98, in preparation.

[25] P. Cho, M. Misiak and D. Wyler, Phys. Rev. **D54** (1996) 3329.